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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Solar Observatory of the Carnegie Institution of Washington

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NOVEMBER 1916

A RELATION BETWEEN THE CONVERGENCE WAVE-LENGTHS IN SPECTRAL SERIES AND THE RADII OF THEIR RESPECTIVE ATOMS, AS COMPUTED FROM EINSTEIN'S PHOTO-ELECTRIC EQUATION AND BY OTHER METHODS — *Fernando Sanford* 201

PRELIMINARY EVIDENCE OF INTERNAL MOTION IN THE SPIRAL NEBULA MESSIER 101 — *A. Van Maanen* 210

ON THE RELATION BETWEEN LINES OF THE SAME SPECTRAL SERIES — *W. M. Hicks* 229

AN EXPERIMENTAL STUDY OF A THEORY OF THE COMPLEX ZEEMAN EFFECT — *A. E. Becker* 236

THE NATURE OF THE CONSTANT-ERROR TERM K' — *C. D. Perrine* 244

ON THE ORBITS OF THE SPECTROSCOPIC BINARIES α ORIONIS AND α SCORPII — *Joseph Lunt* 250

ON THE ORBIT OF THE SPECTROSCOPIC BINARY σ PUPPIS — *Joseph Lunt* 260

ERRATA — 263

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A RELATION BETWEEN THE CONVERGENCE WAVE-LENGTHS IN SPECTRAL SERIES AND THE RADII OF THEIR RESPECTIVE ATOMS, AS COMPUTED FROM EINSTEIN'S PHOTO-ELECTRIC EQUATION AND BY OTHER METHODS[1]

By FERNANDO SANFORD

It seems to be very definitely established that electrons which are discharged from metals under the stimulus of ultra-violet light have, after escaping from the metal, velocities such that their kinetic energies are given by the Einstein equation, $\frac{1}{2}mv^2 = h\nu - p$, where h = Planck's constant, ν = the vibration frequency of the exciting light and p = the work required to remove the electron from the metal. Before leaving the metal the energy of a discharged electron must have been $\frac{1}{2}mv^2 = h\nu$.

It is hard to think of an electron as having, while associated with an electropositive sub-atom, any independent velocity except a vibrational or an orbital velocity about the electropositive sub-atom. If an electron be revolving in an elliptical or circular orbit about its positive sub-atom, it seems probable that in some circumstances its velocity may be accelerated by light vibrations

[1] Read at the meeting of the Pacific Division of the A.A.A.S., at San Diego, California, August 10, 1916.

of its own frequency until its centrifugal force exceeds the centripetal force due to the attraction of its positive nucleus, in which case it will escape as a free electron, with a rectilinear velocity only slightly less than its orbital velocity at the instant of escape.

If this represents the conditions of photoelectric discharge, and if the electrons before their escape were moving in elliptical or circular orbits with orbital velocities such that $\frac{1}{2}mv^2=h\nu$, the Einstein equation gives us a means of computing the mean orbital radius of such electrons. Thus,

$$v^2=\frac{2h\nu}{m} \text{ and } v=\sqrt{\frac{2h\nu}{m}}.$$

The velocity of a particle moving in a circular orbit is $v=2\pi R\nu$. Equating these two values of v and solving for R, we have

$$R^2=\frac{h}{2\pi^2\nu m}$$

$$h=6.57\times 10^{-27}$$

$$m=10^{-27}$$

$$2\pi^2=19.72$$

Hence $R^2\nu=0.333$.

If we use instead of ν, λ, the wave-length of the exciting light, we have $R^2=1.1\lambda\times 10^{-11}$.

This equation would indicate a different electronic radius for each separate line in the spectrum, and would tell us very little about the—so-called—true atomic radius or the radius of the central positive atom about which the electrons are assumed to revolve. It is well known, however, that in a considerable number of elements there are certain series of spectral lines whose wave-lengths may be computed from more or less simple mathematical formulae. All of these series converge toward a shortest possible wave-length, which we may assume to be the shortest wave-length which could exist in the system to which the vibrating electrons belong. If the electrons which produce the spectral lines of the series are moving in orbits about a central positive nucleus, then, apparently, no electron in the system can be closer to the central nucleus than the electron which would give rise to the convergence wave-length. For pur-

poses of comparison we may accordingly assume that the orbital radius corresponding to the convergence wave-length represents the radius of the central positive mass outside of which all the electrons of the system are revolving.

Unfortunately for our comparison, only a few atomic radii have been calculated. A very few molecular radii have been computed from the kinetic gas theory, and Heydweiller has computed the relative diameters of a number of atoms by adopting the assumption of the authors of the Lorenz-Lorentz refraction formula that the ratio $\frac{n^2-1}{n^2+2}$ expresses the ratio of the true volume to the apparent volume of an atom. He has then assumed a diameter for the

TABLE I

Element	Heydweiller Radii	Cube Root of Volume	*R* from Einstein Equation	
			Principal Series	Subordinate Series
Hydrogen	0.85	2.18	1.5	2.0
Lithium	0.87	2.40	1.59	1.96
Sodium	0.96	2.87	1.63	2.12
Potassium	1.19	3.57	1.77	2.24
Rubidium	1.31	3.83	1.81	2.28 2.30
Caesium	1.46	4.13	1.87	2.33 2.37
			Triplet Series	
Magnesium	1.00	2.40	.....	1.66
Calcium	1.21	2.94	.....	1.80
Strontium	1.31	3.27	.....	1.87
Zinc	1.19	2.09	.....	1.59 1.60
Cadmium	1.35	2.35	.....	1.62 1.64
Copper	1.19	1.91	.....	1.87 1.86
Silver	1.37	2.17	.....	1.90 1.87

hydrogen atom based upon the determinations of the molecular diameter calculated from the kinetic gas theory, and has made this his basis for calculating the actual diameters of the atoms. We may also assume that the cube roots of the apparent gram-atomic volumes are approximately proportional to the atomic radii, especially within a group of similar atoms, without making the

Lorenz-Lorentz correction. By comparing the atomic radii calculated by these methods with the radii given by Einstein's equation we may, perhaps, gain further evidence as to the probability of an orbital motion on the part of radiating electrons. Whether this evidence is regarded as important or not, we shall, at least, be

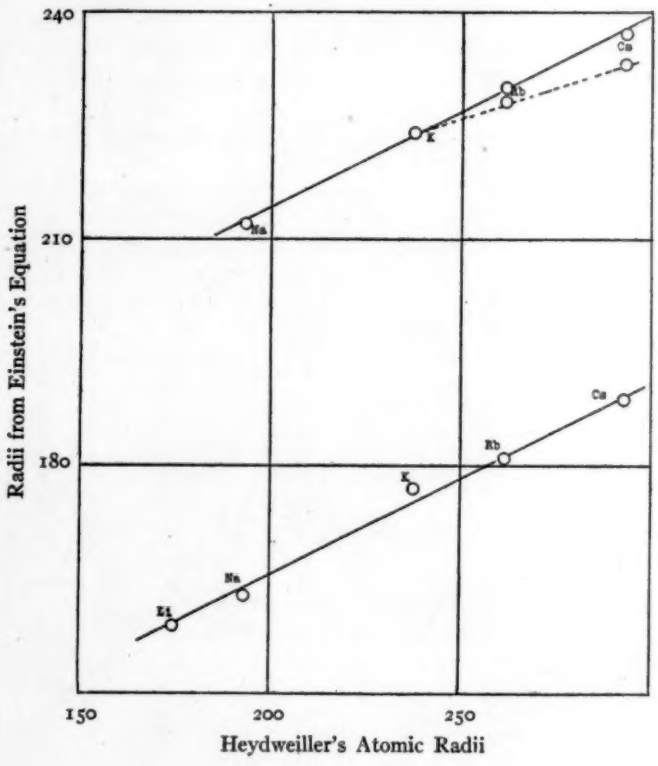


FIG. 1

able to show a relation between the convergence wave-length of a spectral series and a true physical constant of the respective atoms, viz., the radius. This is shown in Table I, in which the calculated radii are multiplied by 10^8.

It is seen from Table I that the radii calculated from the Einstein equation are approximately twice as great as those computed by Heydweiller and are not strictly proportional to them.

Within each group of similar elements, however, this proportionality is very close. This fact is shown graphically for the alkali elements in Fig. 1.

It can also be shown that the Heydweiller radii are, within similar groups, very closely proportional to the square roots of the convergence wave-lengths. In fact, the Heydweiller radii within individual groups can be calculated as accurately from the convergence wave-lengths as from Heydweiller's original data. Thus, for the first subordinate series the radii of sodium, potassium, rubidium, and caesium may be calculated from the equation $R=\frac{\sqrt{\lambda}-508}{137\cdot5}$, as is shown in Table II.

TABLE II

Element	R from $\sqrt{\lambda}$	Heydweiller's R
Na	0.953	0.965
K	1.207	1.19
Rb	1.324	1.131
Cs	1.454	1.465

The Heydweiller radii for magnesium, calcium, and strontium may be calculated from the equation $R=\frac{\sqrt{\lambda}-304}{230}$ with the agreement shown in Table III.

TABLE III

Element	R from $\sqrt{\lambda}$	Heydweiller's R
Mg	1.	1.005
Ca	1.2	1.215
Sr	1.3	1.31

The relation of the Heydweiller radii to the square root of the convergence wave-length is shown graphically for the subordinate series of the alkali metals in Fig. 2. The relations of the atomic radii as calculated from the Einstein equation and the cube roots of the atomic volumes are shown for three spectral series in Figs. 3 and 4.

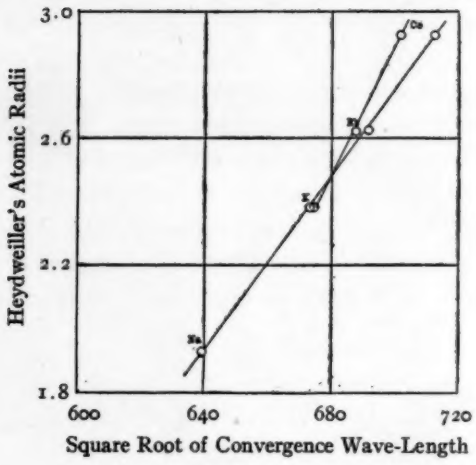

3.0
2.6
2.2
1.8
600
640
680
720
Heydweiller's Atomic Radii
Square Root of Convergence Wave-Length

FIG. 2

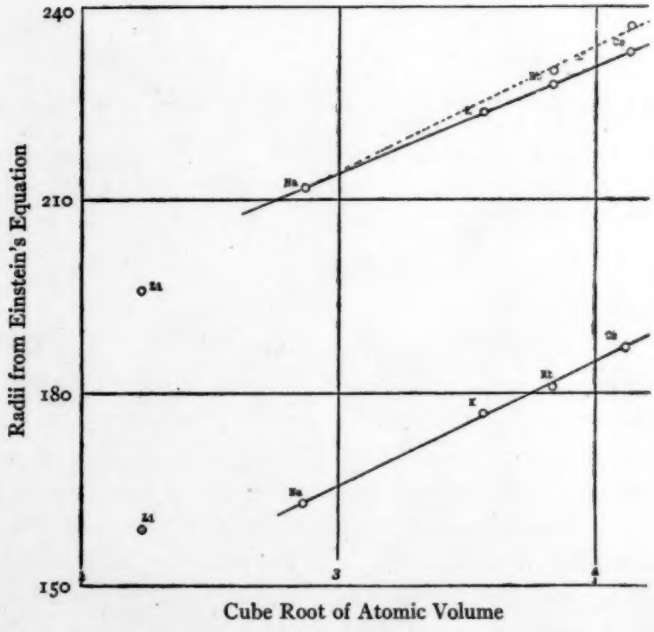

240
210
180
150
3
Radii from Einstein's Equation
Cube Root of Atomic Volume

FIG. 3

The foregoing considerations seem to show that, whether the radiating electrons are revolving in circular orbits about their positive nuclei or not, the Einstein equation gives us a method of calculating an orbital radius which within several groups of similar elements is closely proportional to the respective atomic radius.

Another relation between the atomic radius and the constants used in computing wave-lengths may be found in the Ritz formula. In *Annalen der Physik*, **12**, 265, 1903, Ritz gave a presentation of his theory of serial spectra and proposed a formula which has since

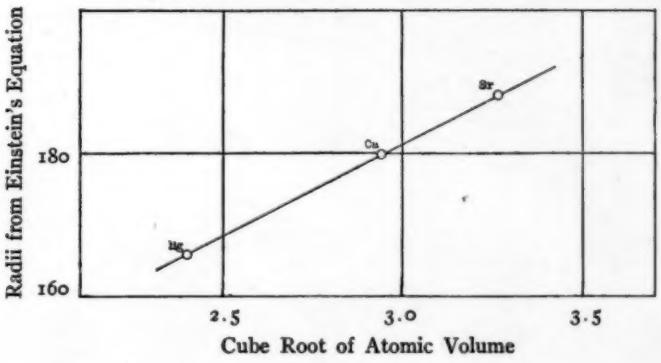


FIG. 4

been shown to give extremely close results in the alkali metals, helium and hydrogen. This formula in its simplest form is

$$\nu = A - \frac{N_0}{\left(n + a + \frac{b}{n^2}\right)^2},$$

where A is the convergence number, N_0 is the Rydberg universal constant and a and b are constants to be determined for each series.

In the article above mentioned Ritz gives his values of these constants for several series, including both the principal series and the two subordinate series of the alkali metals. In an article in *Astrophysical Journal*, **32**, 212, 1910, Birge has used the Ritz formula very successfully with the principal series of the alkalies, the Balmer series in hydrogen and a series of twelve lines in helium. In Fig. 5 Ritz's and Birge's values of a are plotted against the

atomic radius computed from the Einstein equation, and in Fig. 6 the same values of a are compared with the cube root of the respective atomic volume.

It will be seen that in both cases the agreement is remarkably close, and that there can be no reasonable doubt that the Ritz constant a is a function of the atomic radius. Already in his original paper Ritz detected a seeming relation between a and the atomic volume, but the variation was great enough to show that he had selected a wrong quantity for comparison.

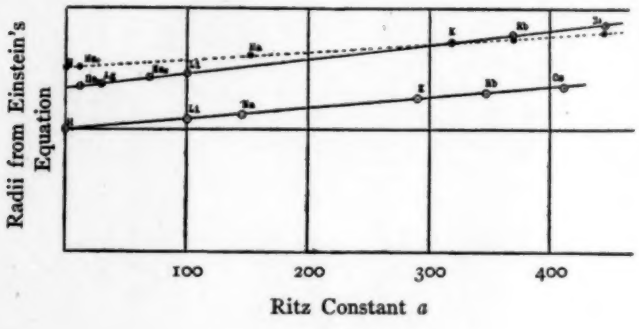


FIG. 5

The curves in Figs. 5 and 6 also give us some information about the distribution of the two subordinate series in the alkali group. They seem to show that the sodium series belong to one subordinate group and the lithium series to the other, as appears in all the curves where the radii of these series are plotted against some other constant. The Balmer series in hydrogen seems to belong to the subordinate group containing the sodium series. Two other hydrogen series having their origins at $A=48764$ and $A=48744$ (as given by Konen) seem to belong to the group of the principal alkali series. The two helium series having their origins at $A=32031$ and $A=29221$ seem to belong to the group of subordinate series containing the lithium series, while the series originating at $A=27173$ seems to belong to the sodium group. A series in silver originating at $A=31542$ also seems to belong to the lithium group. Neither Ritz nor Birge gives a value of a for copper. I have not compared the above-mentioned series to see whether they

agree in their physical properties with the foregoing classification, and I give it here only as suggested from the curves mentioned.

Fig. 6, in which the Ritz constant is plotted against the cube root of the atomic volume, enables us to estimate the atomic volumes of hydrogen and helium. From these curves the cube root of the atomic volume of hydrogen would be at 2.18 or 2.20, according to

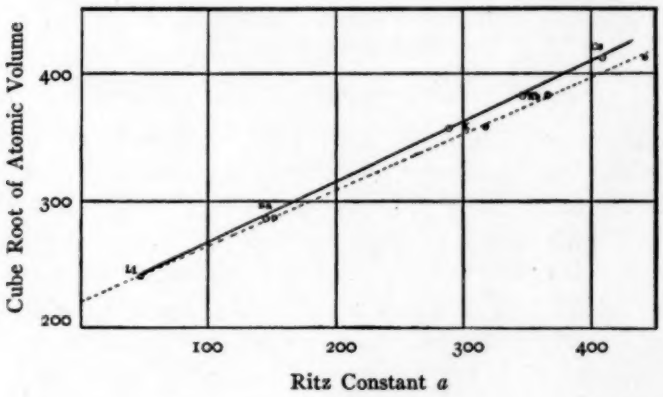


FIG. 6

which atomic system is used, and the cube root of the helium volume would be 2.24 or 2.60, according to which system is considered.

It is probably useless to raise the question at the present time as to whether the different electronic systems which give rise to different spectral series belong to the same atom, or to different atoms; but to me the indications that they actually belong to different atoms seem very strong.

LELAND STANFORD JUNIOR UNIVERSITY
August 19, 1916

PRELIMINARY EVIDENCE OF INTERNAL MOTION IN THE SPIRAL NEBULA MESSIER 101[1]

PHOTOGRAPHS BY
G. W. RITCHEY, J. E. KEELER, C. D. PERRINE, AND H. D. CURTIS
MEASURED AND DISCUSSED BY
A. VAN MAANEN

INTRODUCTION

Although the nebulae have been under observation for many years, it is only within the last quarter-century that their motions have been detected. In 1887 Dreyer[2] stated that "so far we do not possess any clear evidence of change of form or change of place." Several of the older observations seemed to indicate such changes, but Dreyer's careful investigation of all known cases led him to the conclusion that these motions must be illusory "unless we are to believe that nebulae in the good old days moved about as they liked, but have been on their good behavior since 1861 and kept quiet."[3]

Motions of nebulae in the line of sight were first observed with the spectroscope by Keeler in 1890,[4] and since that time the radial velocities of many of them have been determined by Campbell, Slipher, and others; but we remained in ignorance as to their cross-motions for another twenty-five years. Curtis, from a comparison of plates taken with the Crossley reflector of the Lick Observatory by Keeler, Palmer, Perrine, and himself, published in 1915 the proper motions of about one hundred nebulae.[5]

In recent years evidence of internal motion has also been discovered. Here too the radial motions were detected first. In 1914, Buisson, Fabry, and Bourget demonstrated the existence of internal motions in the Orion nebula,[6] a result confirmed by Frost

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 118.

[2] *Monthly Notices*, **47**, 413, 1887. [3] *Ibid.*, **47**, 418, 1887.

[4] *Publications of the Lick Observatory*, **3**, 217, 228.

[5] *Publications of the Astronomical Society of the Pacific*, **27**, 214, 1915.

[6] *Comptes Rendus*, **158**, 1017, 1914; *Astrophysical Journal*, **40**, 241, 1914.

and Maney.[1] In 1914 V. M. Slipher[2] published the "Detection of Nebular Rotation," relating to the Virgo nebula, which is of the "spindle" type, and Wolf[3] detected the rotation of Messier 81, while in 1915 Campbell and Moore[4] announced the rotation of the planetary nebula N.G.C. 7009. Also Pease[5] discovered a possible internal motion in Messier 33.

In December 1915 Mr. Ritchey suggested that the two plates of Messier 101 taken by him with an interval of about five years be placed in the stereocomparator to see if they would show motion. Since no motion whatever was revealed in that way, I then proposed that the plates be placed at my disposal for measurement, to which Mr. Ritchey kindly consented. As sixteen nebulous points chosen for this purpose showed some evidence of internal motion, I at once decided to take up the work in a more extended way, measuring many more points. Feeling also the necessity of measuring at least one other pair of plates, I asked Dr. Curtis if he would be willing to place at my disposal the Crossley reflector plates of this nebula. This request was very kindly granted, and Dr. Curtis sent me three plates, one by Keeler (1899), one by Perrine (1908), and one by himself (1914). The measures on these three plates and on the two made by Mr. Ritchey are discussed in the following pages. My thanks are due to Dr. Campbell, director of the Lick Observatory, and to Dr. Curtis for the use of the Lick plates and for permission to publish my measures of them, and also to Mr. Ritchey for the use of the two plates taken by him.

MEASURES AND DISCUSSION

The measures were made with the "blink" arrangement of the stereocomparator in the manner described in "The Photographic Determination of Stellar Parallaxes with the 60-Inch Reflector."[6] The plates were measured in four pairs, combined and arranged in the stereocomparator as shown in Table I.

[1] *Popular Astronomy*, **23**, 485, 1915. [2] *Lowell Observatory Bulletin*, No. 62.

[3] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **49**, 162, 1914.

[4] *Harvard College Observatory Bulletin*, No. 591.

[5] *Publications of the Astronomical Society of the Pacific*, **28**, 33, 1916.

[6] *Mt. Wilson Contr.*, No. 111, 4 ff.

The exposure times of the Lick Observatory plates were as follows: 1899, June 8, 4^h; 1908, July 28, $1^h\ 45^m$; and 1914, March 20, 2^h; the last plate being under-exposed on account of clouds. The exposure times of the Mount Wilson plates were: 1910, March 10 and 11, $7^h\ 30^m$ (aperture 54 inches); 1915, May 14, 15, and 16, $8^h\ 37^m$ (aperture 50 inches).

TABLE I

Pair	Left Plate-Carrier	Right Plate-Carrier	Observer	Remarks
R I.....	1910	1915	Ritchey, Ritchey	.....
R II.....	1915	1910	Ritchey, Ritchey	.....
L I.....	1899	1908	Keeler, Perrine	1899 through glass
L II.....	1914	1899	Curtis, Keeler	1914 through glass

As the 1899 plate of the Lick Observatory was taken with the early arrangement of the Crossley reflector, involving the use of a Newtonian flat, while the later photographs were made without this auxiliary mirror, the field of the 1899 plate is inverted with respect to that of the later photographs. In using the stereo-comparator it was thus necessary to measure through the glass of one of the plates of the Lick pairs.

The interchange of the new and old plates in the plate-carriers should in the mean eliminate any relative displacement of the images due to the measuring instrument.

Thirty-two stars, as near to the nebula as seemed safe, were used for comparison purposes. It is true, of course, that some of these objects may really belong to the nebula, but the results show that the choice of comparison stars, on the whole, was a fortunate one. To prevent the appearance of a magnitude error it was necessary to avoid a great range in brightness, and in order to increase as much as possible the weight of the plate-constants the comparison stars had to be distributed as uniformly as possible around the nebula. The co-ordinates of the comparison stars relative to the center of the nebula are given in the second and third columns of Table II, + indicating east and north for x and y, respectively.

In the choice of points in the nebula which were to be measured I tried to avoid stars. A priori no certain distinction can be made, but in general I avoided those points which through total absence of surrounding nebulosity give the impression of stars rather than of parts of the nebula. This is a hazardous procedure, and undoubtedly some of the points do not belong to the nebula. From countings on the outer parts of the plates the conclusion was reached that the area containing the 87 measured points should include about 20 measurable stars, but several of these must have been omitted in choosing the points in the nebula. Further, the points selected must be as symmetrical in appearance as possible to avoid difficulties in bisecting their images. This requirement, too, could not be rigorously fulfilled.

TABLE II

CO-ORDINATES OF THE COMPARISON STARS

	x	y		x	y
a	−10′.9	+ 4′.9	*q*	+10′.0	+ 2′.4
b	−10.3	− 0.1	*r*	+10.2	+ 4.7
c	− 6.4	+ 3.4	*s*	+ 8.4	+ 9.1
d	− 6.6	− 8.1	*t*	+ 4.9	+ 5.9
e	− 6.2	− 6.8	*u*	+ 3.3	+ 5.3
f	− 3.8	−10.9	*v*	+ 9.0	+12.0
g	− 3.4	− 8.6	*w*	+ 6.3	+12.0
h	− 0.3	−10.7	*x*	+ 2.3	+11.0
i	+ 1.4	− 5.4	*y*	+ 1.0	+12.6
j	+ 1.8	− 7.1	*z*	− 0.2	+ 8.9
k	+ 2.3	− 9.4	α	− 0.6	+ 6.7
l	+ 7.3	−10.9	β	− 3.8	+ 7.8
m	+ 7.5	− 7.5	γ	− 2.0	+11.5
n	+ 4.7	− 4.7	δ	− 5.9	+11.1
o	+ 6.5	− 2.0	ϵ	−10.5	+ 8.4
p	+ 8.0	+ 0.1	η	− 5.2	+ 5.9

On R I and R II, 87 nebular points were measured; on L I, 69; and on L II, 46. The co-ordinates of these points are given in the second and third columns of Table VI.

Each pair of plates was measured in four positions with east, west, north, and south, respectively, in the direction of increasing readings of the micrometer screw. In the reductions the two measures of each pair in right ascension were combined into one set; the same was done for the measures in declination.

As the measurement of a pair of plates in one position required several hours, the temperature of the room was read during every set of measures. In no case did the range during the measures in one position exceed 1° C. Disturbances from this source are therefore negligible. Moreover, several measures were duplicated to see if any shift could be detected, but no positive evidence was found.

The mean of the measured differences of every object for the two positions of the plate which differ by 180° may be called D. From the values of D the proper motions in α and δ were derived by a method analogous to that described in my paper on stellar parallaxes.[1] It was found from a preliminary reduction, however, that the quadratic terms of the co-ordinates could not here be neglected. The equations of condition therefore have the form

$$D=\mu_\alpha+a+bx+cy+dx^2+exy+fy^2$$

with a similar equation for declination.

Supposing μ_α and μ_δ of the comparison stars to be zero, we can solve the 32 equations for these stars to find the 6 plate-constants a, b, c, d, e, and f. This was done by the regular least-squares method. The constants found for each pair of plates were then substituted into the equations of condition for the stars and for the points in the nebula, thus giving μ_α and μ_δ, the proper motions in α and δ relative to the mean motion of the comparison stars.

These values of μ_α and μ_δ, still expressed in two-thousandths of a revolution of the micrometer screw, must be multiplied by 1.04, 1.04, 0.84, and 0.52 for R I, R II, L I, and L II, respectively, to give the annual proper motion expressed with $0''.001$ as a unit. The resulting values are entered in Table III. The first column contains the designation a, b, ... α, β, ... for the comparison stars, and the numbers 1, 2, 3, ... 87, for the nebular points.

The weight to be given to each pair of plates depends on the scale (which for the Mount Wilson plates is 1 mm = $27''.20$, and for the Lick plates 1 mm = $38''.73$), their quality, and the interval in years. As the influence of quality is difficult to determine in

[1] *Mt. Wilson Contr.*, No. 111, 7 ff.

TABLE III

Total Annual Proper Motions

(Unit = 0".001)

No.	μ_α				μ_δ				Mean	
	R I	R II	L I	L II	R I	R II	L I	L II	μ_α	μ_δ
a	+20	−19	+ 8	+ 2	+24	−26	+24	+21	+ 3	+11
b	+14	− 8	−53	+ 5	−10	−17	−40	+10	−11	−14
c	−28	+ 9	−13	−16	−17	+16	− 4	+16	−12	+ 3
d	+33	+14	+46	+42	−19	+ 4	− 5	−18	+34	−10
e	−10	+18	+ 8	− 2	− 1	+ 9	− 2	+33	+ 4	+10
f	−18	−27	−23	−18	+15	− 4	+ 4	− 7	−22	+ 2
g	+ 5	+22	+18	− 2	−16	−14	−12	− 6	+11	−12
h	− 8	− 7	+ 5	−10	0	+ 8	+18	−18	− 5	+ 2
i	−35	−19	−27	−32	+41	+25	+30	+13	−28	+27
j	−12	− 4	− 3	−18	+10	+14	+10	− 5	− 9	+ 7
k	+ 1	+12	+14	+ 9	− 5	−20	+20	−14	+ 9	− 5
l	−19	−15	−44	− 9	+ 9	+17	+24	+17	−22	+17
m	+52	+31	+34	+39	+ 8	−48	−60	+ 9	+39	−23
n	+26	+ 7	+ 7	+20	−28	+27	−52	+26	+15	− 7
o	− 3	−11	−13	−29	−38	+ 3	+21	−27	−14	−10
p	+ 5	+16	− 5	+ 1	− 3	−10	−10	+ 2	+ 4	− 5
q	+10	− 9	+41	+ 1	+11	+14	+23	−15	+11	+ 8
r	−50	+12	− 6	+ 2	−32	− 5	+ 5	+ 9	−11	− 6
s	−21	−43	+16	+13	+33	+18	+ 1	+ 8	− 9	+15
t	+14	+ 5	− 4	− 2	+15	−13	+ 3	−17	+ 3	− 3
u	+24	+11	+ 2	+ 8	− 6	−18	− 1	−22	+11	−12
v	− 6	−10	−40	−36	+25	+31	+10	−12	−23	+14
w	+33	+22	+21	+20	−21	−25	−16	+14	+24	−12
x	−11	−12	−13	− 5	−14	+19	+26	− 2	−10	+ 7
y	+18	+ 7	0	+ 6	−26	−29	−18	+ 9	+ 8	−16
z	+26	+ 8	+ 6	− 6	−51	−43	− 8	+12	+ 9	−23
α	0	− 4	+ 3	+ 8	+52	−10	+17	+ 6	+ 2	+16
β	−56	− 6	−46	−11	+46	+64	+21	+14	−30	+36
γ	− 1	+ 6	+13	+12	+14	−23	−35	+13	+ 8	− 8
δ	−10	− 7	+22	+10	−16	+40	− 3	−14	+ 4	+ 2
ϵ	−10	+15	+16	−21	−12	+ 4	+18	−30	0	− 5
η	+25	− 1	+22	+ 8	+12	−14	+ 5	−32	+14	− 7
1	+43	+32	.....	.....	− 1	+12	.....	.....	+38	+ 6
2	−20	−23	−55	− 9	+ 8	+16	0	+12	−27	+ 9
3	+14	− 4	.....	.....	−16	+50	.....	.....	+ 5	+17
4	+16	− 3	+45	+ 3	+50	+50	+24	− 5	+15	+30
5	+ 7	− 4	+ 5	+ 3	+ 6	+42	− 2	+21	+ 3	+17
6	+ 7	+ 1	+68	.....	+40	+24	+87	.....	+25	+50
7	+26	+17	.....	.....	+ 4	+58	.....	.....	+22	+31
8	+14	+ 1	+15	−15	+18	+48	+15	+ 7	+ 4	+22
9	+35	+23	+45	+37	−18	− 7	−34	−49	+35	−27
10	+14	−10	+35	+15	+ 6	+33	+34	+ 6	+14	+20
11	− 6	+34	+ 3	.....	+ 4	+56	−24	.....	+10	+12
12	−22	+19	+61	.....	−30	+14	− 6	.....	+19	− 7
13	− 2	+ 4	+ 1	− 9	+10	+20	+ 5	+15	− 2	+13
14	+14	+ 4	+ 8	+ 3	− 2	+15	+23	+ 4	+ 7	+10
15	+38	+17	+23	+ 2	− 2	+35	+21	+28	+20	+21
16	+47	+25	+42	+12	− 1	+12	+34	+26	+32	+18

TABLE III—*Continued*

No.	μ_α				μ_δ				Mean	
	R I	R II	L I	L II	R I	R II	L I	L II	μ_α	μ_δ
17	+ 7	+25	.....	.....	− 1	+ 9	.....	.....	+16	+ 4
18	+43	+33	+91	.....	−10	− 2	+60	.....	+56	+16
19	+ 9	+16	−12	.....	−24	+ 6	+19	.....	+ 4	0
20	−21	+ 5	+29	.....	−42	+17	+22	.....	+ 4	− 1
21	−12	−43	+18	+26	+22	+ 2	+58	+23	− 3	+26
22	+ 7	− 6	− 8	+ 3	+21	+22	− 8	+ 4	− 1	+10
23	+14	− 1	.....	.....	+ 4	−12	.....	.....	+ 7	− 4
24	+21	+ 4	.....	.....	+18	+54	.....	.....	+13	+36
25	+ 5	+10	− 9	.....	+20	+ 9	+ 1	.....	+ 2	+10
26	−19	−36	.....	.....	−29	− 8	.....	.....	−28	−19
27	+11	−15	−26	−16	−40	− 2	+ 6	+20	−12	− 4
28	+19	+15	+15	.....	−75	− 1	+30	.....	+16	−15
29	+52	− 2	−55	.....	+ 4	−31	−81	.....	− 2	−36
30	− 6	−47	.....	.....	−10	−31	.....	.....	−27	−21
31	+45	− 8	+23	.....	+23	−27	−15	.....	+20	− 6
32	+ 4	−49	+29	−27	−24	+16	+28	+28	−11	+12
33	−20	−28	−22	.....	+16	+ 2	+ 9	.....	−23	+ 9
34	−11	−44	+15	−13	−22	+12	+66	+16	−13	+18
35	+27	−21	−32	−32	− 1	−10	+28	+16	−15	+ 8
36	+ 9	−30	−12	−21	+ 5	+24	− 8	− 9	−14	+ 3
37	−23	−12	.....	.....	− 9	+27	.....	.....	−18	+ 9
38	+23	−23	.....	.....	+19	+ 4	.....	.....	0	+12
39	−25	−36	+13	− 7	+ 5	+ 6	− 8	+27	−14	+ 8
40	−44	−16	.....	.....	−21	−18	.....	.....	−30	−20
41	+24	+16	.....	.....	−12	−19	.....	.....	+20	−16
42	−48	−44	−19	.....	−16	−27	−29	.....	−37	−24
43	+58	−35	.....	.....	−22	−45	.....	.....	+12	−34
44	− 3	+11	−15	.....	−44	−60	+19	.....	− 2	−28
45	+14	−28	+64	.....	+ 8	+ 1	−36	.....	+17	− 9
46	−11	−29	−36	−20	+29	−16	−12	−38	−24	− 9
47	−38	− 9	−15	− 7	+20	−26	−30	−28	−17	−16
48	+ 6	−29	−18	−14	−15	−25	+30	−24	−14	− 9
49	− 9	−57	+30	− 3	−10	−42	− 3	−37	−10	−23
50	+ 5	−43	+37	−16	+ 5	−46	−37	−20	− 4	−25
51	+30	−40	+46	.....	−11	−49	−34	.....	+12	−31
52	−69	−83	+ 5	.....	− 6	−38	−60	.....	−49	−35
53	+50	−17	+ 4	− 1	−14	−28	−39	−12	+ 9	−23
54	−15	−34	−32	−45	−32	−73	−18	−21	−32	−36
55	−49	−77	−19	−28	−36	−11	+21	+21	−43	− 1
56	+47	+ 1	+51	− 1	−45	−66	−19	−27	+25	−39
57	+15	−11	+18	− 4	−49	−57	+11	− 3	+ 5	−25
58	− 5	+10	+59	.....	−51	−52	−44	.....	+21	−49
59	+33	− 6	+24	− 9	−73	−60	−89	−38	+11	−65
60	+25	+ 9	−14	.....	−15	−54	−34	.....	+ 7	−34
61	−14	−35	−10	+ 3	−46	−41	−66	− 2	−14	−39
62	+ 7	+ 1	+44	−20	−52	−51	−19	−16	+ 8	−35
63	+21	−20	− 7	− 9	−74	−60	−46	−21	− 4	−50
64	+15	−57	+76	−28	−77	−11	+41	− 5	+ 2	−13
65	+28	−29	+19	− 7	−58	−38	−23	−31	+ 3	−38
66	+60	−28	+23	−27	−30	−22	−44	−41	+ 7	−34
67	+58	− 4	+ 8	− 6	−48	−31	−31	−48	+14	−40

TABLE III—*Concluded*

No.	μ_α				μ_δ				Mean	
	R I	R II	L I	L II	R I	R II	L I	L II	μ_α	μ_α
68.....	+ 1	−10	+69	.....	−59	−69	−66	.....	+20	−65
69.....	+ 7	−28	.....	.....	−46	−29	.....	.....	−11	−38
70.....	+15	+24	+24	+10	−70	−55	−25	−27	+18	−44
71.....	−11	−21	−23	+11	+12	−28	−39	+10	−11	−11
72.....	+21	−15	+18	− 3	−42	−46	−13	−12	+ 5	−28
73.....	+38	− 1	+33	.....	−59	−72	−14	.....	+23	−48
74.....	+11	− 4	.....	.....	−107	−55	.....	.....	+ 3	−81
75.....	+12	+23	+ 5	.....	−35	−18	−33	.....	+13	−29
76.....	+41	+27	.....	.....	−38	−34	.....	.....	+34	−36
77.....	+29	+37	+115	.....	−17	−57	−10	.....	+60	−28
78.....	+37	+33	+13	+13	+ 1	−33	−33	−36	+24	−25
79.....	+41	+ 9	+24	− 3	−28	−68	−71	−38	+18	−51
80.....	+14	+18	+37	− 3	−21	−41	− 6	−35	+17	−26
81.....	+ 2	−19	+34	− 5	+ 5	−61	+24	−29	+ 3	−15
82.....	+ 4	+46	+42	− 5	−34	−75	−14	−24	+22	−37
83.....	+31	+ 5	+20	+ 6	−80	−63	−31	+ 4	+16	−43
84.....	+81	+28	+56	.....	+ 2	+11	+71	.....	+55	+28
85.....	+16	+34	.....	.....	−17	+ 3	.....	.....	+25	− 7
86.....	−17	+20	+47	+16	−11	+36	− 7	+18	+17	+ 9
87.....	+43	+31	.....	.....	−33	+31	.....	.....	+37	− 1

advance, all pairs were provisionally given the same weight and combined to form mean motions for the 32 comparison stars and the 46 nebular points which were measured on all four pairs.

The average deviation from these means for each pair of plates was as shown in Table IV (unit = 0″.001). Since the mean deviations

TABLE IV

Pair	R I	R II	L I	L II
Comparison stars.....	12	11	11	13
Nebular points.....	16	16	17	14

are sensibly the same for the different pairs, it seems permissible to use the same weights for all pairs, the quality very nearly counterbalancing the influence due to scale and time interval. The adopted mean μ_α and μ_δ are accordingly those given in the last two columns of Table III. There is of course quite a range in the agreement for the different points, but the mean probable error of a mean μ_α or μ_δ is only 0″.008.

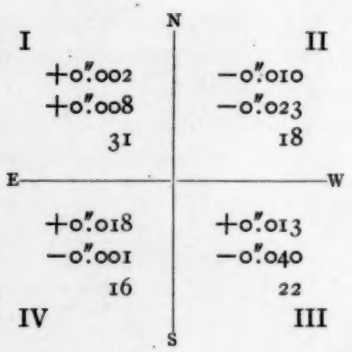
The results found may be due partly to a translation of the nebula as a whole, and partly to internal motions. To derive the motion of translation of the nebula three methods were employed.

1. The mean of the proper motions μ_α and μ_δ for all the nebular points was found.

This gives $\mu_\alpha = +0''.005$, $\mu_\delta = -0''.012$. The Mount Wilson plates treated separately give: $\mu_\alpha = +0''.002$, $\mu_\delta = -0''.016$, while the Lick plates alone give $\mu_\alpha = +0''.007$, $\mu_\delta = -0''.008$.

2. Whatever the character of the internal motions of a nebula like Messier 101, we may expect them to be nearly symmetrical with respect to the center of the nebula. If, therefore, we divide the points into four groups corresponding to the four quadrants, the two pairs of opposite quadrants should give a good agreement. Using all the 87 points for this purpose, the results are as summarized in Diagram I, which gives for each quadrant the mean values of μ_α and μ_δ, and the number of points measured.

DIAGRAM I

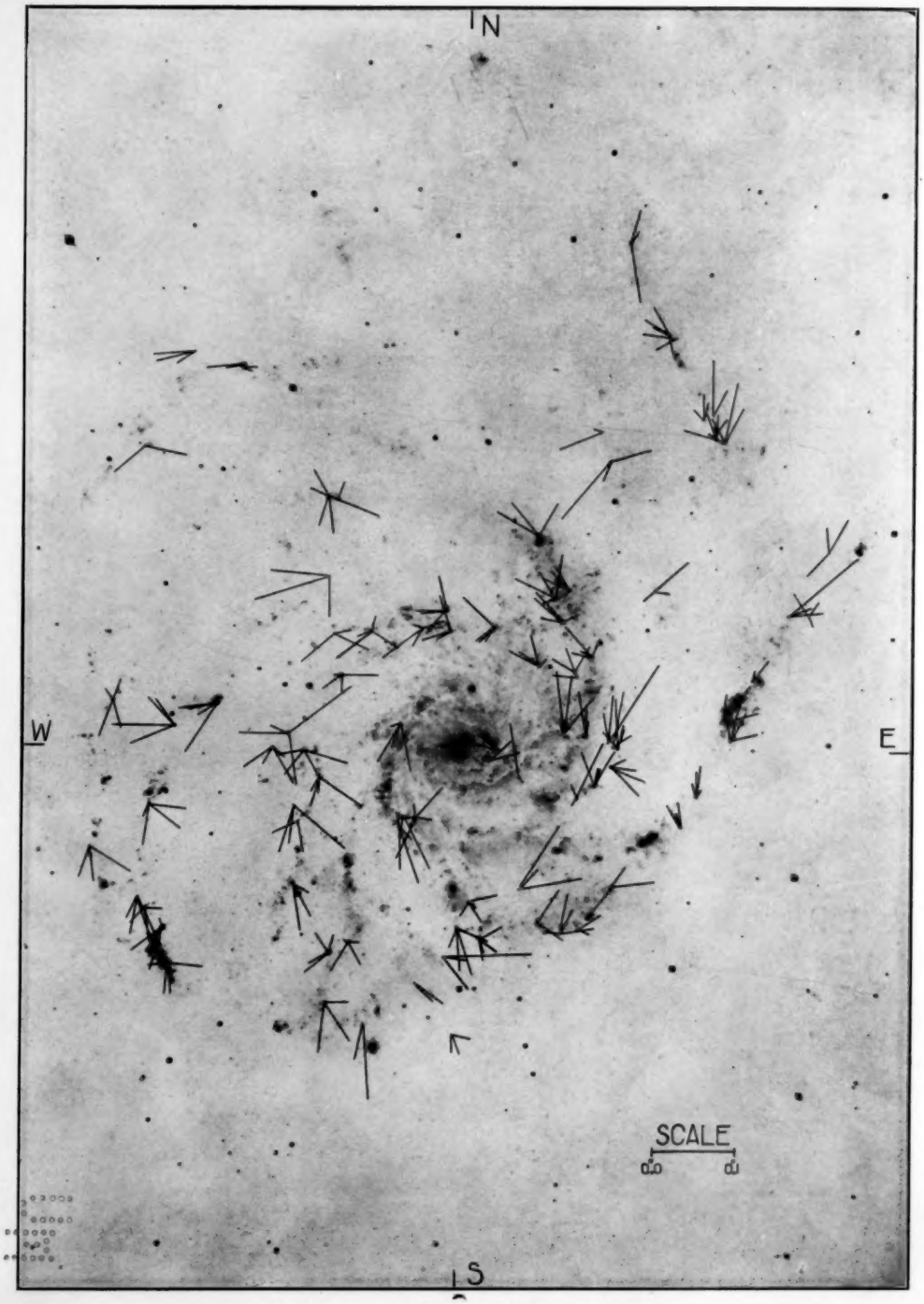


Mean of I and III $\begin{cases} \mu_\alpha = +0''.007_5 \\ \mu_\delta = -0''.016 \end{cases}$

Mean of II and IV $\begin{cases} \mu_\alpha = +0''.004 \\ \mu_\delta = -0''.012 \end{cases}$

3. As, however, the distribution of the points measured is far from uniform in the four quadrants, it seemed worth while also to derive analogous values for the central parts of the nebula, within a radius of 5'.0, where the distribution of the points is more symmetrical. The results may be seen in Diagram II.

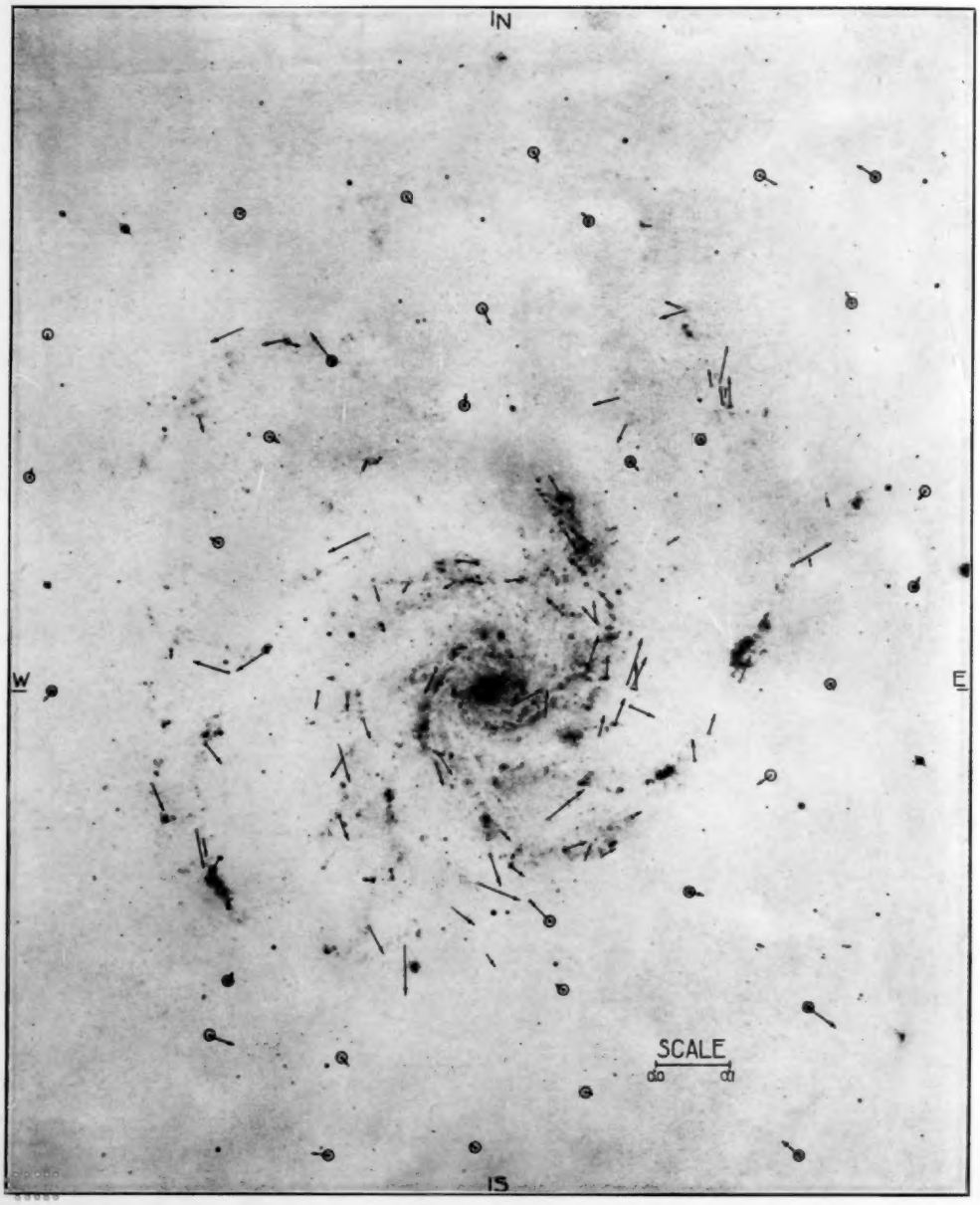
The results of the three methods are given in Table V. The agreement throughout is as good as could be expected. For the final motion of translation the values $\mu_\alpha = +0''.005$, $\mu_\delta = -0''.013$



INTERNAL MOTIONS IN MESSIER 101

The lines indicate the direction and magnitude of the annual motions of the nebulous points as derived from the different pairs of plates. Their scale (0″.1) is indicated on the illustration. The scale of the nebula is 1 mm = 9″.1.

PLATE VII

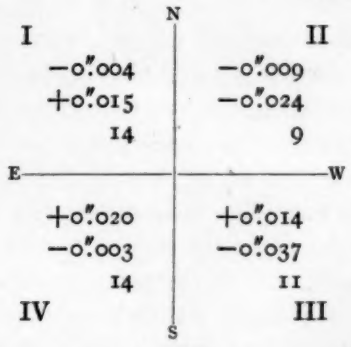


INTERNAL MOTIONS IN MESSIER 101

The arrows indicate the direction and magnitude of the mean annual motions. Their scale (0″.1) is indicated on the illustration. The scale of the nebula is 1 mm = 10″.5. The comparison stars are inclosed in circles.

have been used. No great weight, however, should be given to this motion of translation, as the hour angles and the parallax factors of the different plates may have vitiated the results. With

DIAGRAM II



Mean of I and III $\begin{cases} \mu_\alpha = +0''.005 \\ \mu_\delta = -0''.011 \end{cases}$

Mean of II and IV $\begin{cases} \mu_\alpha = +0''.005^5 \\ \mu_\delta = -0''.013^5 \end{cases}$

the material at hand, however, it is the best we can do. Subtracting the adopted μ_α and μ_δ from the total motions in Table III, the results are what may be called the internal motions. They are given in the fourth and fifth columns of Table VI.

TABLE V

Method	μ_α	μ_δ
1.....	+0″.005	−0″.012
2.....	+0.006	−0.014
3.....	+0.005	−0.012

The accompanying illustrations show these motions, freed from the motion of translation of the whole nebula. The density of the center of the prints used for the engravings was reduced in order to show the motions more clearly. Plate VI indicates the motions given by each pair of plates; they show in general a good agreement, although there are several large discrepancies. Plate VII shows the mean annual proper motion for each of the 87 points, and also for the comparison stars, which are surrounded by circles. The scale for the annual motions is indicated on the plates.

If the results as illustrated in Plate VII could be taken at their face value, they would certainly seem to indicate a motion of

rotation or possibly motion along the arms of the spiral. Without expressing a final opinion as to the character of the motion, which must be determined by future work, it may be of interest to examine the evidence afforded by the existing material.

To discuss the internal motions from the standpoint of rotation, they were analyzed into components along and perpendicular to the radius; the latter for convenience will be spoken of as the rotational component. The results are given in the last four columns of Table VI. In view of the uncertainty of the measures it seems well to treat the available material in two ways: first, by taking all points measured, and, secondly, by excluding those points which, because of internal disagreement of the measures on the four sets, seem to be uncertain in their final motion. As the probable error in μ_α or μ_δ derived from a single pair of plates is a little over 0".013, I have thought it well to exclude in the latter case points whose deviation in μ_α or μ_δ for one or more of the pairs of plates is >0".040. In this way the following 25 points were excluded: Nos. 6, 11, 12, 18, 20, 28, 29, 34, 35, 43, 44, 45, 49, 50, 51, 52, 53, 64, 66, 67, 68, 77, 81, 83, and 84. This procedure seems the more reasonable as 18 of these 25 points are involved asymmetrically in the nebulosity; the measures in such cases must be very uncertain if the plates compared are not of exactly the same density.

The results are as follows, numbers in parentheses relating to the restricted group: 78 (55) points have a left-handed motion,[1] only 9 (7) moving right-handedly; 58 (44) points appear to be moving outward, while 28 (17) show motion inward. The rotational motion is the larger in the majority of cases, viz., for 63 (45) points. The mean rotational motion is 0".022 (0".021) left-handed; the mean radial motion 0".007 (0".009) outward.

The probable reality of the result is indicated by the satisfactory agreement of the pairs of plates as shown in Table VII, where the + sign indicates left-handed and outward motions, respectively. For this comparison only the 46 objects common to all plates were used.

[1] The rotation directions refer to the illustrations. See Plate VII, noting the orientation.

TABLE VI

Co-ordinates and Internal Motions

(Unit = 0″.001)

No.	x	y	μ_α	μ_δ	$\mu_{rad.}$		$\mu_{rot.}$	
					Out	In	L.H.	R.H.
1.....	+0″.7	−0″.3	+33	+19	21	.....	32	.....
2.....	+0.9	−0.1	−32	+22	.....	34	20	.....
3.....	+1.3	−0.6	0	+30	.....	13	27	.....
4.....	+2.3	+0.6	+10	+43	20	.....	39	.....
5.....	+2.8	+0.3	− 2	+30	1	.....	30	.....
6.....	+3.2	+0.1	+20	+63	22	.....	62	.....
7.....	+3.4	0.0	+17	+44	17	.....	44	.....
8.....	+3.5	0.0	− 1	+35	.....	1	35	.....
9.....	+3.3	−0.4	+30	−14	31	.....	.....	11
10.....	+3.0	−0.8	+ 9	+33	0	0	34	.....
11.....	+2.6	−1.1	+ 5	+25	.....	5	25	.....
12.....	+3.4	−3.0	+14	+ 6	7	.....	14	.....
13.....	+4.8	−1.7	− 7	+26	.....	15	22	.....
14.....	+5.2	−1.0	+ 2	+23	.....	2	23	.....
15.....	+5.9	+0.2	+15	+34	17	.....	33	.....
16.....	+5.7	+0.6	+27	+31	30	.....	28	.....
17.....	+6.3	+1.5	+11	+17	15	.....	14	.....
18.....	+7.1	+2.9	+51	+29	58	.....	8	.....
19.....	+7.6	+2.9	− 1	+13	4	.....	12	.....
20.....	+7.9	+4.3	− 1	+12	5	.....	11	.....
21.....	+5.7	+6.7	− 8	+39	25	.....	31	.....
22.....	+5.4	+6.7	− 6	+23	14	.....	19	.....
23.....	+5.5	+6.9	+ 2	+ 9	8	.....	4	.....
24.....	+5.4	+7.2	+ 8	+49	44	.....	24	.....
25.....	+5.2	+7.1	− 3	+23	17	.....	16	.....
26.....	+4.6	+8.9	−33	− 6	.....	20	17	.....
27.....	+4.4	+9.0	−17	+ 9	1	.....	19	.....
28.....	+3.5	+10.9	+11	− 2	2	.....	.....	11
29.....	+3.1	+6.2	− 7	−23	.....	24	.....	4
30.....	+3.0	+6.8	−32	− 8	.....	20	26	.....
31.....	+4.2	+3.4	+15	+ 7	16	.....	.....	4
32.....	+1.6	+4.6	−16	+25	19	.....	24	.....
33.....	+2.5	+3.4	−28	+22	1	.....	36	.....
34.....	+2.0	+3.2	−18	+31	17	.....	32	.....
35.....	+2.2	+2.8	−20	+21	3	.....	29	.....
36.....	+2.8	+2.0	−19	+16	.....	7	24	.....
37.....	+2.5	+1.5	−23	+22	.....	8	31	.....
38.....	+2.5	+1.6	− 5	+25	9	.....	23	.....
39.....	+1.7	+1.8	−19	+21	2	.....	28	.....
40.....	−4.8	+8.2	−35	− 7	12	.....	34	.....
41.....	−4.9	+8.1	+15	− 3	.....	10	.....	11
42.....	−5.9	+8.4	−42	−11	15	.....	40	.....
43.....	−6.9	+6.5	+ 7	−21	.....	19	11	.....
44.....	−2.9	+5.4	− 7	−15	.....	10	14	.....
45.....	−2.7	+5.3	+12	+ 4	.....	2	.....	13
46.....	−0.3	+3.0	−29	+ 4	7	.....	28	.....
47.....	+0.7	+2.6	−22	− 3	.....	9	20	.....
48.....	−0.2	+2.5	−19	+ 4	5	.....	18	.....

TABLE VI—*Continued*

No.	x	y	μ_α	μ_δ	$\mu_{rad.}$		$\mu_{rot.}$	
					Out	In	L.H.	R.H.
49.....	−0.′9	+2.′5	−15	−10	.....	5	17	.....
50.....	−1.9	+2.5	− 9	−12	.....	4	15	.....
51.....	−2.8	+2.4	+ 7	−18	.....	17	9	.....
52.....	−2.9	+3.6	−54	−22	16	.....	56	.....
53.....	−2.6	+1.6	+ 4	−10	.....	9	6	.....
54.....	−5.2	+0.9	−37	−23	33	.....	29	.....
55.....	−6.2	+0.4	−48	+12	48	.....	.....	8
56.....	−6.7	−1.3	+20	−26	.....	15	29	.....
57.....	−7.5	+0.9	0	−12	.....	2	12	.....
58.....	−8.0	−2.2	+16	−36	.....	6	39	.....
59.....	−6.9	−3.3	+ 6	−52	16	.....	49	.....
60.....	−6.8	−3.6	+ 2	−21	8	.....	19	.....
61.....	−6.4	−4.0	−19	−26	30	.....	12	.....
62.....	−6.2	−4.7	+ 3	−22	11	.....	19	.....
63.....	−1.3	+0.5	− 9	−37	.....	5	38	.....
64.....	−3.7	+0.3	− 3	0	3	.....	0	0
65.....	−4.1	−0.1	− 2	−25	3	.....	25	.....
66.....	−3.4	−0.1	+ 2	−21	.....	1	21	.....
67.....	−3.0	−0.7	+ 9	−27	.....	3	28	.....
68.....	−3.5	−1.3	+15	−52	4	.....	54	.....
69.....	−3.4	−1.6	−15	−25	24	.....	16	.....
70.....	−3.5	−3.0	+13	−31	11	.....	32	.....
71.....	−2.7	−4.5	−16	+ 2	7	.....	.....	15
72.....	−2.3	−4.3	0	−15	13	.....	7	.....
73.....	−2.8	−5.5	+18	−35	23	.....	32	.....
74.....	−2.0	−6.0	− 2	−68	65	.....	19	.....
75.....	−0.1	−6.2	+ 8	−16	16	.....	8	.....
76.....	−0.9	−5.1	+29	−23	17	.....	33	.....
77.....	−0.2	−4.6	+55	−15	12	.....	56	.....
78.....	+0.5	−4.1	+19	−12	14	.....	18	.....
79.....	0.0	−3.9	+13	−38	38	.....	13	.....
80.....	+0.3	−3.3	+12	−13	14	.....	11	.....
81.....	−0.9	−1.5	− 2	− 2	3	.....	.....	1
82.....	−1.2	−1.5	+17	−24	8	.....	28	.....
83.....	−1.3	−1.7	+11	−30	17	.....	27	.....
84.....	+1.4	−3.1	+50	+41	.....	17	63	.....
85.....	+2.6	−3.9	+20	+ 6	6	.....	20	.....
86.....	+2.2	−4.0	+12	+22	.....	13	21	.....
87.....	+1.8	−3.8	+32	+12	2	.....	34	.....

TABLE VII

	$\mu_{rot.}$	$\mu_{rad.}$
R I.....	+0.″021	+0.″004
R II.....	+0.032	+0.012
L I	+0.017	+0.006
L II.....	+0.012	+0.007

Of the comparison stars, 13 have left-handed and 19 right-handed motion; 15 move outward and 15 inward; the mean rotational motion, as well as the mean radial motion, is 0".000; a glance at Plate VII shows that very few of the stars can share in the motions of the nebula.

The measures indicate a small and hardly trustworthy decrease of rotational motion with increasing distance from the center, as shown by Table VIII.

TABLE VIII

Distance	Mean Distance	Rotational Component	Number of Points
<3'.1	2'.2	0".024 (0".026)	19 (14)
3'.1 to 5'.0.....	3.9	0.028 (0.024)	29 (16)
5.1 to 7.0.....	5.9	0.014 (0.015)	18 (15)
>7.0	8.9	0.019 (0.021)	21 (17)

The change in the rotational component with increasing distance from the center is of interest in connection with the assumption that the nebular points are moving about the nucleus in elliptical orbits. For different points the observed orbital motion should then be inversely proportional to the square root of the mean distance from the center of the nebula. It does not seem safe to attempt final conclusions on the basis of the material now available.

The annual rotational component of 0".022 at the mean distance from the center of 5' corresponds to a rotation period of about 85,000 years. If we knew the parallax of the nebula, and if we could assume that the motions and the distances of the points from the center are mean values for elliptical orbits, the central mass could be calculated. The parallax is unknown and the assumption concerning distances and motions is probably far from the truth. Nevertheless, such an assumption, combined with more or less probable values for the parallax, may give a notion as to whether we are dealing with masses of the order of those of the stars or with much greater aggregations of material. It goes without saying that the universality of the Gaussian gravitation constant must also be assumed.

The relation between the parallax and a central mass which would cause a particle to move in an elliptical orbit at a mean distance of 5′ with a period of 85,000 years can be derived from Kepler's third law; it is $M = 0.0037\,\pi^{-3}$. We thus find the corresponding values of parallax and mass given in Table IX.

TABLE IX

π	Mass (Sun=1)	Mean Distance (Astron. Units)	Rotational Compt. km/sec.
0″.0001....	3,700,000,000	3,000,000	1,038
0.0004....	57,812,500	750,000	260
0.0016....	903,320	187,500	65
0.0064....	14,114	46,875	16
0.0256....	220	11,719	4
0.1024....	3.4	2,930	1

The order of the parallax of Messier 101 is suggested by the following considerations: Curtis gives 0″.033 as the average annual motion of 66 large spiral nebulae.[1] Could we assume that these objects are comparable with the stars in the matter of proper motion and distance, we should have, supposing them to be of the twelfth magnitude, 0″.005 for their mean parallax. Again, we may compare the cross-motion of 0″.033 with the observed radial velocities. Only a few of the latter have been determined as yet, but those found indicate high speeds. Assuming them to be distributed at random in space, Curtis finds the distance to be of the order of 10,000 light-years,[2] corresponding to a parallax of 0″.0003. Various objections to the acceptance of these results, even as rough guesses, immediately suggest themselves, but they are the best we have. At the moment, therefore, this method of discussion suggests large values for the masses involved.

An inspection of Plate VII conveys the impression that the general drift of the motion is outward along the branches. The direction of the branches in the vicinity of 52 of the points can be specified with fair accuracy, and a comparison with the observed motions shows a mean divergence for the latter of $7° \pm 4°$ toward the *concave* side of the spiral.

[1] *Publications of the Astronomical Society of the Pacific*, 27, 217, 1915.

[2] *Ibid.*, 218, 1915.

It would be of interest to consider the further development of a nebular mass in which, as apparently is the case with Messier 101, the direction of motion is so closely coincident with the branches of the spirals, and to examine the evidence afforded by other nebulae as to the probability of the continued existence of such motion. For example, continued outward motion should result in forms in which the distribution of the nebular material is widely different from that in Messier 101. Interesting cases such as Messier 77, Messier 81, Messier 101, Messier 100, and possibly N.G.C. 7293 immediately suggest themselves in this connection; in these we see a gradual transition from a nebula with the bulk of the material strongly concentrated in the center, while the spirals contain relatively little material, to the other extreme, where the spirals, containing most of the material, seem to be connected hardly at all with the central body.

All such questions as those touched on above, which naturally are highly speculative, are here considered because of the part played by elliptical motion in the Chamberlin-Moulton hypothesis as to the origin of spiral nebulae. The explanation which they have given, at least for the typical case, requires that the motion of a particle belonging to one of the branches should be inclined at a considerable angle toward the convex side of that branch. The departure of Messier 101 from the typical case imagined by Chamberlin and Moulton is sufficient to excite comment. Professor Chamberlin, however, kindly allows me to quote parts of his manuscript of a book, *The Origin of the Earth*, bearing on this question. He states:

> . . . . that the paths pursued by the projectiles are not identical with the spiral chain of projectiles into which they are forced to arrange themselves. The divergence between the paths and the chain of nebulous matter may vary widely. A much closer approach to coincidence between the paths of the projectiles and the chain of projectiles is assignable in certain cases of closer approach and more violent projection.

And also:

> . . . . the relative amount of this forward or tangential pull is a critical factor; its value is obviously dependent on the relative distance to which the bolt was projected.

The results can therefore perhaps be reconciled with the Chamberlin-Moulton hypothesis, by assigning sufficiently high values to the "disruptive" forces as compared with the central attractions; but all such questions can well await the accumulation of further observational data. In the meantime my measures of two photographs of Messier 81 (made by Mr. Ritchey in 1910 and 1916) give preliminary evidence of internal motion similar to that revealed in the case of Messier 101.

For the precision of such results the time factor is of course the most important, but when the photographs are to be used for the study of internal motions, various precautions will also contribute in no small degree:

1. Several photographs of each object should be taken; 4 or 5 old plates compared with 4 or 5 new ones should enable us to derive the motions satisfactorily.

2. Care should be taken that both hour angles and parallax-factors are similarly distributed in both the old and the new series of plates. A neglect of this precaution undoubtedly will introduce errors difficult or impossible of elimination; some of the discrepancies in the measures on Messier 101 are probably due to this cause.

3. As it is impossible to measure points near the center, if the exposures are long enough to show the fainter parts of a nebula, two sets of plates of the same object are desirable, one with long, the other with short, exposures.

4. The plates should be taken with a telescope of the greatest possible focal length, in order to increase the scale.

5. It is advisable to insert a star-trail on at least one plate for purposes of orientation. This of course is not necessary in deriving possible internal motions, but for the motion of translation it will afford more accuracy.

6. For a determination of the parallaxes of spiral nebulae, the distribution of the plates, if feasible, should be as outlined in my paper on stellar parallaxes.[1] The short exposures used for the proper motions might be utilized in part for this work.

[1] *Mt. Wilson Contr.*, No. 111.

7. On account of asymmetry in the nebulous points, the plates should be measured in a purely differential manner, so that the settings for any point can be made on both plates in quick succession. The chance of choosing different parts of the image for the settings on the two plates is thereby greatly decreased. The stereocomparator is an admirable apparatus for such measures, but if not available, two miscroscopes may be used on an ordinary measuring machine, adjusted in such a way that they point to the same part of the field on the two plates, which are mounted side by side on the plate-carriage.

To test the possibility that the results found might be a consequence of systematic personal or instrumental errors, it was important that the plates should be measured with another machine and, if possible, by another measurer. I am glad to acknowledge the great service rendered by Mr. Seth B. Nicholson, who willingly spent a large amount of time in remeasuring parts of the Mount Wilson plates with two different machines. We first mounted a second microscope alongside the first, on one of our regular instruments used for measuring spectra. The two plates were mounted on the plate-carriage moved by the micrometer screw in such a way that the microscopes were directed toward nearly identical points on the two plates. In this way we were able to bisect corresponding points one after the other. As the instrument allowed only a part of a plate to be measured without readjustment on the carriage, 8 or 9 points were selected on each side of the center of the nebula, and their relative shift in declination was derived with respect to a dozen comparison stars. Mr. Nicholson found an annual difference of motion in declination of 0".083 (the east side of the nebula moving northward as compared with the west side), while my measures with the stereocomparator had given for the same points a relative shift in the same direction of 0".063. The agreement is satisfactory.

After having thus proved that the motion found was not due to any defect of the stereocomparator, Mr. Nicholson kindly undertook with this instrument a set of measures of 53 points of the nebula, using all the 32 comparison stars for reference. The plates were measured in all four positions and reduced in exactly the same way

as the original measures. Of the 53 points measured, 11 are points mentioned as difficult on p. 220. The results for all 53 points, and also for the 42 remaining after excluding these 11 difficult points, are summarized in Table X.

TABLE X

Annual Motions	53 Points	42 Points	Agreement of Sign with Van Maanen	
			53 Points	42 Points
μ_α of translation	+0".003	+0".002	72%	76%
μ_δ of translation	−0.013	−0.011	75	83
$\mu_{rad.}$	−0.003	−0.002	68	69
$\mu_{rot.}$	+0.009	+0.010	75	76

The agreement of the motion of translation with that given on p. 218 is very satisfactory. Although Mr. Nicholson's value of the rotational component is smaller than that found above, a comparison with the data on p. 222 shows that the difference is within the uncertainty of the determination.

I wish to express my thanks to Mr. Hale and Mr. Seares for many suggestions they have made during the work on the plates discussed here, and to Miss Helen Davis for much assistance in the computations.

Mount Wilson Solar Observatory
March 19, 1916

ON THE RELATION BETWEEN LINES OF THE SAME SPECTRAL SERIES

By W. M. HICKS

The very accurate determination of the wave-lengths of the first 57 members of the principal series of sodium published by Messrs. Wood and Fortrat in a recent number of this Journal (**43**, 73, 1916) affords data which help to throw some light on the question as to the nature of the formulae giving series lines, or as to whether in fact such formulae actually exist. For the measures a degree of accuracy involving errors not exceeding a few thousandths of an angstrom is claimed, even up to within the last few members—a claim which is probably well justified. There can be no doubt but that the frequencies of any of the recognized series can be approximately given by a formula of the type $n=A-\frac{N}{[f(m)]^2}$ and I have shown that, with the exception of a few cases only, the form $f(m)=m+\mu+\frac{d}{m}$, suggested by Mogendorff[1] and myself[2] as a modification of Rydberg's, gives the frequencies to within a few hundredths, except in many cases for $m=1$, and that it is in fact better than the Ritz formula used by Wood and Fortrat. In general the constants are determined from the lines for $m=2, 3, 4$, for which of course the calculated and observed values agree, but even so their values will be subject to uncertainty, due to the observational errors of the three lines. When we come to lines beyond $m=4$, the observational errors are usually so considerable that no conclusion is possible beyond the fact that any given formula can or cannot represent the observations approximately. The limit A of the series is determined with considerable accuracy, in nearly all cases from the lines for $m=2, 3, 4$. This in itself is a remarkable result. The actual calculated value of A, however, not only suffers from the errors of observation of the three lines,

[1] *Proc. Roy. Acad. Amsterdam*, **9**, 434, 1906. [2] *Phil. Trans.*, A, **210**, 57, 1909.

but also depends to some extent on the form of $f(m)$ employed. For instance, the value of the wave-numbers may differ by a few units—or say 1 in 10,000—according as the formula a/m, or Ritz's β/m^2 is used. It might be thought that in the case where many members of a series are known the value might be best obtainable by considering those of the highest order. But this is not the case, because not only are the higher orders faint and subject to large observational errors in wave-length, but being in general in the ultra-violet, these errors are multiplied by large factors in the wave-numbers. For instance, in the case of Na, the limit is close to 2400 A.U., or a wave-number about 41450. An error of 0.1 A.U. is multiplied by 17.2, whilst errors in λ considerably larger than 0.1 may be expected. The great importance of Wood and Fortrat's measurements is that even up to $m=54$ the observations are probably correct to within a few thousandths of an A.U. Therefore we may use the last half-dozen preceding this to calculate asymptotically the value of A, and thus obtain the true numerical value of $f(m)$ for any line, independent of any supposition as to its general form, and only affected with the single observational error of the line itself. For the lines of high order an ordinary Rydberg formula is accurate to the degree of approximation attainable.

In the numerical work which follows I have used Wood and Fortrat's wave-lengths given in international units and reduced them to vacuo by means of the table given by Kayser in his *Handbuch*, **2**, 514, the numbers in the tables being increased by $\frac{1}{60}$ to render them applicable to wave-lengths in air at 15° C. instead of 20° C. For Rydberg's constant I have used $N=109679.2$, the value on the international scale deduced from the careful measures by W. E. Curtis[1] of the spectrum of H. For the determination of the limit the formula $f(m)=m+1.148$ is sufficient. Using this and any value of the limit obtained in the ordinary way, the more accurate value 41449.00 was found. The frequencies calculated from $41449.00+\xi-\frac{N}{(m+1.148)^2}$ for $m=49.....54$ exceed the observed by respectively $-0.01, +.00, -.03, -.03, +.01, +.00,$

[1] *Proc. Roy. Soc.*, A, **90**, 605, 1914. This agrees very closely with Rydberg's value on the Rowland scale, reduced to the international, viz., 109678.6.

or an average of $\xi - 0.01$. An error of 0.01 may arise as much from residual errors of calculation in the last significant figures as from any other cause and will make no change in λ up to 0.001 A.U. and only a few units in the sixth decimal place of $f(1)$. The formulae for P_1 and P_2, using this limit and the first and third lines (the second is affected with a possible observational error of $\lambda = 0.030$), are then

$$P_1 \qquad n = 41449.00 - \frac{N}{\left\{m + 1.148088 - \frac{.031215}{m}\right\}^2}$$

$$P_2 \qquad n = 41449.00 - \frac{N}{\left\{m + 1.147256 - \frac{.031126}{m}\right\}^2}.$$

The values of $d\lambda$ = calculated − observed wave-lengths are given under H in the table on p. 232. The corresponding values deduced from Wood and Fortrat's paper are given under W. F. The wave-numbers used are given in the second column.

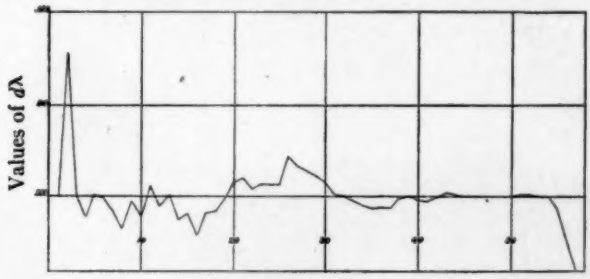
The deviations shown by the three last lines from their calculated values are so large that they can scarcely belong to the typical P series. The lines from $m = 44$ to 54 agree exactly with their calculated values. Of the three in question the first and third agree with Ritz combinations, $s(1) - d(55)$ and $s(1) - d(56)$, where s, d denote respectively the sharp and diffuse sequences, $s(1)$ being the limit $P(\infty)$. Their corresponding differences from the observed lines are respectively -0.001 and $+.002$. It is not clear what relation the second has to known sequences. It differs from the calculated combination $s(1) - s(55)$ by -0.013, so that this allocation seems doubtful. They suggest that in the ionised vapor the molecular or atomic configurations which would give diffuse or sharp series up to $m = 54$ do not exist, but have all transformed to p configurations. Above $m = 54$, however, they have in this case not been so further transformed, and the corresponding p configurations are in consequence absent; above $m = 56$ not even these are formed and no further systems capable of absorbing light of p, s, or d frequencies exist. It would be interesting to know whether a different treatment as to heat or the presence of other ionising

sources in the vapor would modify the last few lines observable in all cases, whatever the number of lines showing.

m	n	H.	W.F.	m	n	H.	W.F.
1.....	16956.16	0.000	0.014	26.....	41300.92	0.043	0.042
	16973.34	.000	.000	27.....	41311.13	.033	.032
2.....	30267.38	.172	.141	28.....	41320.39	.028	.027
	30272.87	.157	.306	29.....	41328.71	.022	.021
3.....	35040.18	.000	−.076	30.....	41336.20	.015	.012
	35042.66	.000	.000	31.....	41342.90	.002	−.001
4.....	37296.23	−.035	.032	32.....	41349.18	.000	−.004
	37297.73	−.021	−.027	33.....	41354.87	−.004	−.007
5.....	38540.10	−.040	−.005	34.....	41360.07	−.009	−.012
	38541.58	.002	−.018	35.....	41364.85	−.012	−.015
6.....	39298.47	−.022	−.002	36.....	41369.31	−.012	−.014
	39299.26	−.002	−.014	37.....	41373.53	−.012	−.010
7.....	39793.66	−.075	−.010	38.....	41377.39	−.002	−.005
	39794.96	−.014	−.061	39.....	41380.95	.000	−.002
8.....	40136.75	−.034	−.022	40.....	41384.15	−.004	−.006
9.....	40383.17	−.005	.003	41.....	41387.18	−.005	−.006
10.....	40565.65	−.020	−.015	42.....	41390.05	−.002	−.002
11.....	40705.74	.011	.022	43.....	41392.78	.003	.002
12.....	40814.13	−.010	−.008	44.....	41395.21	.001	.000
13.....	40900.92	.002	.004	45.....	41397.50	.000	.000
14.....	40970.46	−.025	−.022	46.....	41399.66	.000	−.002
15.....	41027.95	−.019	−.026	47.....	41401.69	.000	.000
16.....	41075.21	−.042	−.046	48.....	41403.57	.001	−.003
17.....	41115.59	−.019	−.021	49.....	41405.40	.000	.000
18.....	41149.53	−.016	−.018	50.....	41407.08	.000	.000
19.....	41178.72	−.003	−.005	51.....	41408.70	.002	.002
20.....	41203.97	.015	.014	52.....	41410.20	.002	.003
21.....	41225.71	.019	.018	53.....	41411.58	.000	−.001
22.....	41244.42	.008	.014	54.....	41412.94	.000	.000
23.....	41261.13	.013	.014	55.....	41413.99	−.013	−.014
24.....	41275.75	.012	.014	56.....	41414.63	−.046	−.047
25.....	41288.77	.012	.009	57.....	41415.24	−.079	−.079

As to the other lines, the table shows clearly that if the observational errors do not amount to more than a few thousandths of an angstrom, neither formula is sufficient to the degree of accuracy here in question, although that in α/m gives slightly better results than that in β/m^2. To produce differences of the magnitude shown, for instance, between $m=14$ and 36, the values of the denominators $f(m)$ must differ very largely from those given by the formulae, and the formulae give the results so closely as they do because m is so large and only the first few digits in $f(m)$ are required. Messrs. Wood and Fortrat point out that the deviations appear to show signs of periodicity. This can best be exhibited graphically. In the accompanying diagram the abscissae are values of m and the

ordinates of $d\lambda$. Two phenomena are clear at a glance. If regard be had to the fact that there will be some disturbance in the curve for low values of m, because the deviations for $m=1$ and 3 are forced to be zero, there would appear to be a mean set of deviations, going through a period of about 16 in m. Superposed on these is another variation alternately positive and negative which is almost certainly real. Such cannot be represented by any simple algebraic function, although the latter effect might be represented by a term $a(-)^m$ in $f(m)$. It would seem that if $f(m)$ is a function of m alone, circular or transcendental functions are required.



But it is possible that $f(m)$ may not be a function of m. If the various frequencies in a series all arise as different modes of vibration of the same configuration, we should expect such a function to exist. If, however, each line comes from a special rearrangement of the components of an atom, this might no longer happen. For instance, with the same m (say m electrons absent) a number of rearrangements might exist, each capable of giving different spectral lines. That something of this nature may be a reality is indicated by the existence of what I have called lateral displacement.[1] These depend on multiples of the oun.[2] It would seem that the atom could not adjust itself to the exact conditions given by a function $f(m)$, but as if it could only get as near as possible by a number of discrete changes. Just as the shape of a brick wall can be adjusted

[1] "A Critical Study of Spectral Lines," Part III, *Phil. Trans.*, A, 213, 336, 1913.

[2] The word "oun" is used by the author to designate $90.47\,w^2$, where $w = \frac{\text{atomic weight}}{100}$.—Eds.

to a curve $y=f(m)$ only roughly by abrupt steppings of the individual bricks, or a multiple of bricks. Here the bricks are the ouns. In any case it is clear that in a diffuse series of strong lines and satellites, the two sets cannot both be determined by functional forms of $f(m)$. In a large number of elements, the corresponding denominators $f(m)$ of the two sets (strong and satellite) differ by one number of ouns for $m=1$, and by a number different from this but the same for other values of m.[1] Moreover, it is possible in diffuse series in all cases to express the denominators so that the differences of their successive decimal parts differ by an integral number of ouns. It is interesting to see whether a similar rule is applicable to the principal series of Na. In sodium the oun is given by $4\delta_1=\delta=19.17$. Δ, which gives the doublet separation, is $743=155\delta_1$. The quantities are so small that their values can only be determined to three significant figures, and therefore it is not possible to draw any convincing conclusions. If, however, the fourth significant figure in Δ is 7, so that $\Delta=743.7$, $\delta=19.19$, the observed differences do appear to come very close to such multiples. Taking the observational errors in λ to be p in the third decimal place (p probably <2), these denominators are given in the following list from $m=1$ to 7. The numbers in parentheses denote maximum possible errors, the third column gives their differences, whilst the oun multiples are given on the right.

m.			
1.	2.116873(0)		
2.	3.132683(38)	**15810**	$21\Delta+10\delta=15809.6$
3.	4.137683(3.8p)	**5000**	$6\Delta+28\delta=4999.5$
4.	5.140000(7.6p)	**2317**	$3\Delta+5\delta=2327.0$
5.	6.141977(16p)	**1977**	$2\Delta+25\delta=1967.1$
6.	7.142810(25p)	**833**	$\Delta+5\delta=839.6$
7.	8.143087(39p)	**277**	$15\delta=287.8$

The differences 2317, 1977 differ by 10 or $2\delta_1$, in opposite directions from the multiples given. It may be due to an error of 0.0013 A.U. in the observation for $m=4$. This discussion would have been wholly fanciful were it not for the very accurate measurements of wave-length. But, as is seen, even now, the possible observational errors themselves are comparable with δ, even for

[1] *Phil. Trans.*, A, **210**, 72, 1910; **213**, 347, 1913; e.g., in Cd.

$m=3$, so that no certain conclusion is possible. The cases of K, Rb, Cs are, however, different, the values of δ for these being respectively 55.45, 263.77, 638.22. One main object of the present note is to draw attention to the particular importance of using the same instrument in Weiss' laboratory to extend observations to these elements. Bevan has shown that Wood's method is applicable to all the alkalies as well as to Na.

Even with Na, however, it is possible to draw an important conclusion with certainty. It refers to a curious irregularity in the differences of the denominators for corresponding lines in the P_1 and P_2 series. As these depend on close lines any observational errors are probably of about the same amount and produce little effect on their differences. I have shown[1] that whilst in many elements these differences for $m \gtrless 2$ may be the same and considerably less than for $m=1$, in the alkalies the values for $m \gtrless 2$ are larger. The present measures enable us to test this question for six intervals with very considerable exactitude. The result is as follows (taking as above $\Delta=743.7$).

m		
1.	743	$\Delta=155\delta_1$
2.	769	$\Delta+5\delta_1=40\delta=767.7$
3.	802	$\Delta+3\delta=$ 801.3
4.	928	$\Delta+38\delta_1=$ 926.1
5.	1563	$2\Delta+4\delta=$ 1564.1
6.	1313	$2\Delta-9\delta=$ 1314.6

The agreement is close, especially when it is remembered that errors of 1 or 2 units in the last place may be introduced as residuary errors due to calculations with seven-place logarithms of numbers of 7 significant figures. Again, however, on account of the smallness of δ, we can feel no absolute certainty as to the multiples of the oun. But the changes, the large change for $m=5$ and return to a smaller for $m=6$, must certainly be real, unless the list contains misprints. Both series can scarcely be represented by any simple algebraic function of m.

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[1] *Phil. Trans.*, A, **212**, 33, 1912.

AN EXPERIMENTAL STUDY OF A THEORY OF THE COMPLEX ZEEMAN EFFECT

By A. E. BECKER

INTRODUCTION

Various attempts have been made to account for the complex Zeeman effect. Preston's[1] suggestion that it may be due to complexity in the parent line has since been restated by Drude[2] and Bohr.[3] Wali-Mohammed[4] also suggests that the thallium line λ 5351 is really double, as it gives six Zeeman components which can be symmetrically arranged in two triplets. Apparently he does not extend the idea to other complex types. On the other hand, Paschen[5] has shown that narrow series doublets (or triplets), whose components give complicated magnetic effects for low fields, may combine into a single normal Zeeman triplet at high fields. The writer, however, has been led by his investigations on series spectra to an explanation identical with the suggestion of Preston. Even in the face of Paschen's work it seems worth while to present the results obtained by such a study of the complex Zeeman effect.

The fundamental hypothesis is that the complex Zeeman effect is due to complexity in the structure of the parent line, which is supposed to consist of a number of components of equal or nearly equal frequency. Each of these gives rise to a Zeeman triplet of normal or nearly normal type. It is assumed that the number of components in the parent line equals the number of p-components (i.e., Zeeman components polarized in a plane perpendicular to the lines of force). Each of the latter is accompanied by two s-components (i.e., Zeeman components circularly polarized about the lines of force), exactly as in the simple effect. Such a triplet will be called a reduced triplet. It is not maintained that the p-components are necessarily at the positions of the parent-line

[1] *Dublin Society Transactions*, **6**, 385, 1898; *Philosophical Magazine*, **45**, 331 1898; *Nature*, **59**, 224, 1899.

[2] *Theory of Optics*, p. 447.

[3] *Philosophical Magazine*, **27**, 518, 1914.

[4] *Annalen der Physik*, **39**, 248, 1912.

[5] *Ibid.*, **39**, 897, 1912.

components, for in that case change of field-strength would not affect their positions, which is contrary to experimental fact. The inference is that the number of p-components is the same as the number of components in the parent line, and vice versa. Thus the application of the hypothesis leads to an investigation of the two propositions:

1. In any complex Zeeman effect the number of s-components is always twice the number of p-components.

2. These Zeeman components may be arranged in symmetrical reduced triplets, one for each p-component, of normal or nearly normal interval.[1]

APPLICATION OF THE HYPOTHESIS

Let us first study the established complex types as illustrated by Voigt,[2] a reproduction being given in Fig. 1. The continuous vertical line in the middle of this diagram indicates the positions of the parent lines. Two parallel lines are drawn at a normal distance, a, one on either side of the central one. The letters p and s denote the p- and s-components respectively. A', A, A''; B', B, B'', etc., indicate respectively the components of the reduced triplets. In any given type these triplets are all of the same interval. This is given in terms of a at the right of the types in Fig. 1.

It is now evident that type I reduces to three triplets, as it has three p-components, A, B, and C, together with the necessary six s-components, A', A''; B', B''; and C', C'', all properly placed. Type III is a triplet. It is obvious that type IV reduces to three triplets; type V, to three; type VIII, to two; type IX, to three; type X, to three; and type XII, to five.

In type VI one encounters difficulties. But Runge and Paschen,[3] who established these various types, seem to have been uncertain of the number of components in this one. It must therefore be regarded as an exception needing further experimental investigation.

Since type II has two p- and four s-components, it satisfies proposition 1. But the reduced triplets are unsymmetrical, the

[1] In this connection the reader will find the article by Ritz (*Annalen der Physik*, **25**, 676, 1908) of importance.

[2] *Magneto- und Electro-optik*, p. 87. [3] *Astrophysical Journal*, **15**, 235, 1902.

interior *s*-components, A'' and B', and the exterior ones, A' and B'', being respectively at intervals $2a$ and $3a/2$ from their *p*-components, A and B. Thus type II is an exception to proposition 2. But

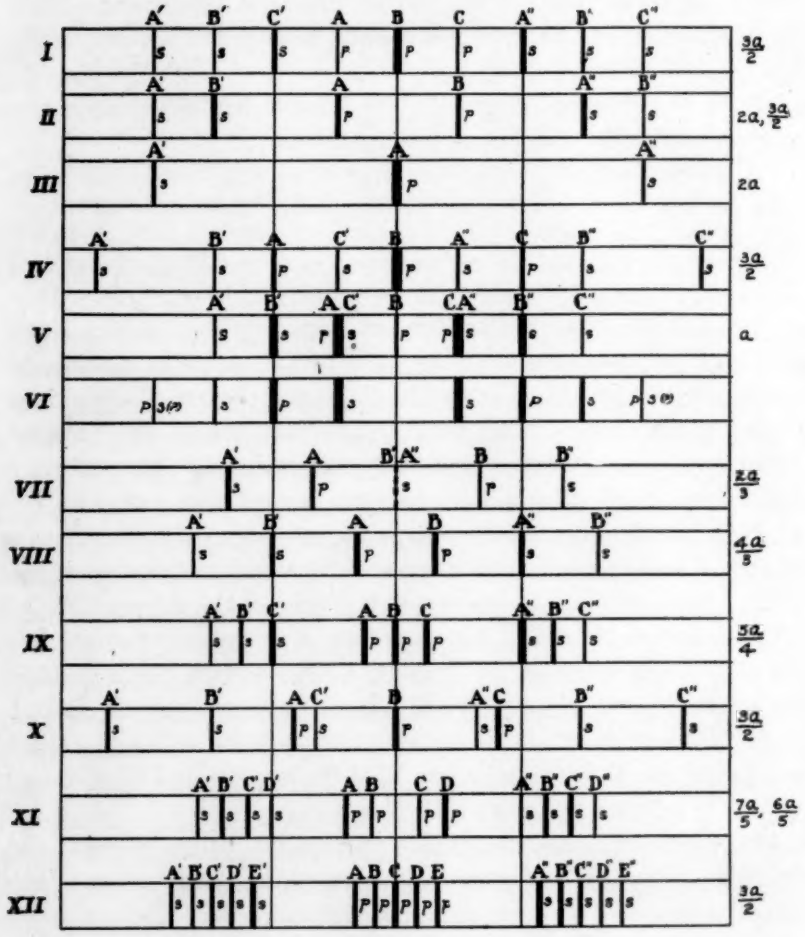


FIG. 1

it is significant that these unsymmetrical displacements are those of the reduced triplets of types III and I respectively; for, as Voigt[1] points out, any three parent lines which give rise to types I, II, and III, respectively, are closely related from the standpoint of series

[1] *Loc. cit.*

spectra. And Paschen[1] shows that series combination terms of the form $(np_2 - mS)$ give Zeeman triplets of interval $3a/2$, which is also significant, since np_2 is taken from the formula for lines giving type II. Similarly, type XI may be regarded as four unsymmetrical reduced triplets, the displacements being $7a/5$ and $6a/5$.

Type VII satisfies neither proposition. But by assuming two s-components, as yet unobserved, at the position of the parent line, it reduces to two symmetrical triplets of interval $2a/3$, which is just half that of those in type VIII. Thus the assumption has merit, since series spectra again show that parent lines giving rise to types VII and VIII are very closely allied. One would thus expect each of these parent lines to be close doublets, which is the conclusion of Michelson[2] and others for the D_1 and D_2 lines.

The assumption of the superposed s-components in type VII affords opportunity to appeal to experiment. Along the lines of force only the s-components are visible. By analogy with the simple effect, the primed s-components of the reduced triplets should be circularly polarized in one direction and the double-primed ones in the other. The interesting case is that in which a circularly right-handed polarized component falls on a circularly left-handed polarized component, as in type VII at the assumed central s-component. Such a double component should be unpolarized when viewed along the lines of force.

We have thus found that of the twelve types of Fig. 1 eight satisfy both propositions 1 and 2; two others satisfy proposition 1; another requires further experimental investigation; and the remaining one points out the method for a beautiful experimental test of the truth of the two propositions.

If the assumed s-component of type VII really exists, it is probably of weak intensity and thus rather unsuitable for examination along the lines of force. However, a study of King's[3] results for iron and titanium shows that several complex types afford opportunity for carrying out the experiment. A convenient type

[1] *Loc. cit.*

[2] *Philosophical Magazine*, **34**, 290, 1892.

[3] King, "The Influence of a Magnetic Field upon the Spark Spectra of Iron and Titanium," *Carnegie Institution of Washington, Publication No. 153*, pp. 22–42.

is that illustrated in Fig. 2, where *a* represents the parent line; *b*, the *p*-components; *c*, the *s*-components at right angles to the lines of force; *d*, the *s*-components which one expects to be circularly polarized in one direction; and *e*, those in the other. That is, one expects the central *s*-component to be unpolarized along the field.

This expectation is verified experimentally, as shown by the writer's photographs, Plate VIII, Figs. 3 and 4. Another type which occurs several times in both spectra is that of Plate VIII, Fig. 5, which again has the central unpolarized *s*-component. It thus

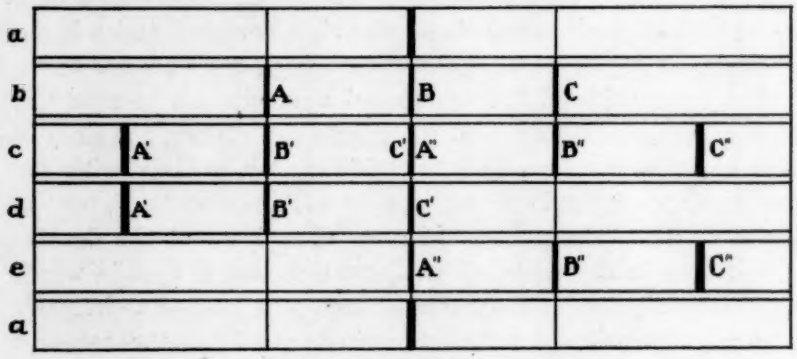
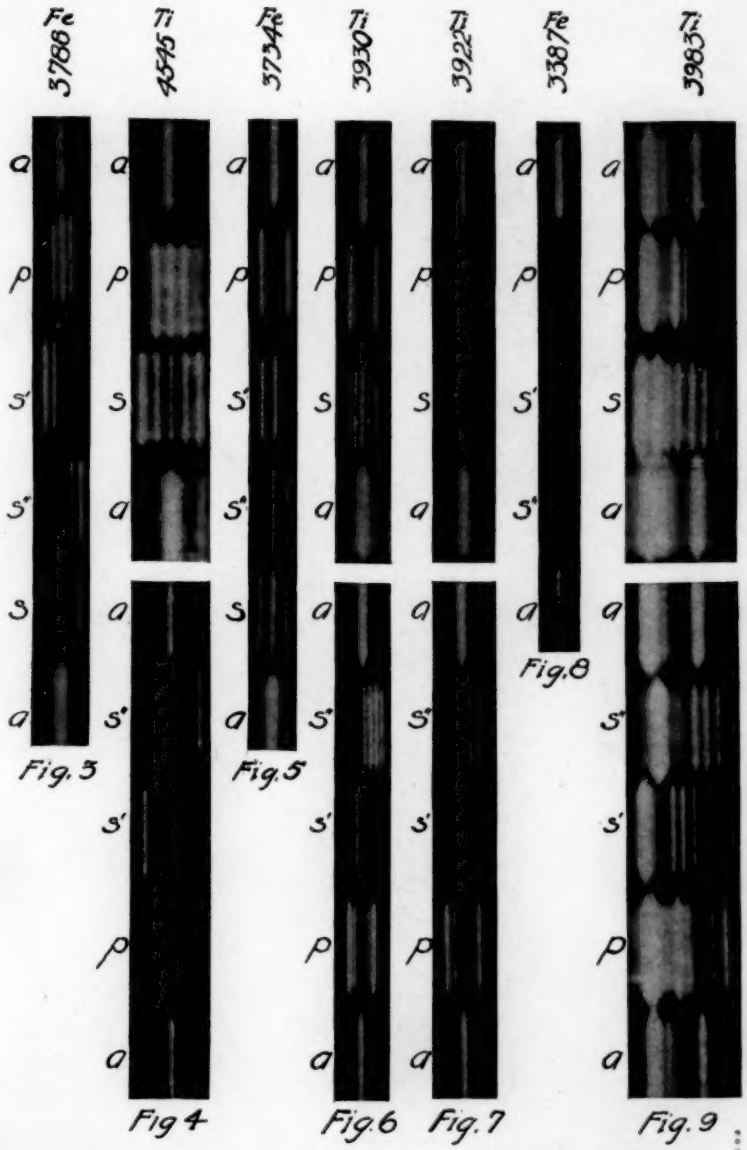


FIG. 2

has four *s*- and two *p*-components, as called for by proposition 1. Fig. 6, on the same plate, presents another type, which is seen to consist of twelve components instead of eleven, as it appears when viewed at right angles to the field. The same is true in Plate VIII, Fig. 7, except that the central *s*-components are not exactly superposed. A further verification of proposition 1 is shown by Plate VIII, Figs. 8 and 9, in each of which there are two of these unpolarized *s*-components, thus giving eight *s*- and four *p*-components. Becquerel and Deslandres[1] had already discovered the unpolarized *s*-component in Plate VIII, Figs. 3 and 5, but the writer was able to predict that there would be two of these unpolarized *s*-components in cases such as shown in Plate VIII, Figs. 8 and 9. There are several other types, especially in titanium, which in the same

[1] *Comptes rendus*, **127**, 18, 1898.

PLATE VIII



a = No magnetic field. p = p-components.
s = s-components. s' = s-components
circularly polarized in one direction.
s' = s-components circularly polarized in
other direction. Field strength = 23,800.
Scale of about 1 Angstrom per cm.

way verify the hypothesis that there are twice as many *s*- as *p*-components in every complex Zeeman effect.

To test the hypothesis in another way, the writer repeated the experiments of King.[1] A field of 23,800 gausses was used, the investigation extending from λ 5050 to λ 3230 for iron and titanium. Some of the complex types found do not agree with King's results. In titanium the region common to both investigations is from λ 5050 to λ 3659, for which King records 165 Zeeman triplets, 23 lines showing complex effects, 1 unaffected line, and 133 lines whose Zeeman effect was doubtful. For the same 322 lines the writer finds 157 Zeeman triplets, 45 lines showing complex effects, 1 unaffected line, and 119 lines of a doubtful number of Zeeman components. Thus the writer finds a larger number of lines of complex Zeeman effect, in which the components are photographically well-defined and sharp, than does King.

Between λ 5050 and λ 3230, the writer finds in titanium 67 lines showing complex Zeeman effects of unquestionable character. Of these lines 62 have twice as many *s*- as *p*-components and therefore satisfy proposition 1. Of the remaining 5, 3 are apparently quartets, 1 an octuplet, and the other a quintuplet having but one *p*-component. This latter may be regarded as two reduced triplets in which the *p*-components for some reason do not separate when the field is applied. There is also the line λ 4296 which is apparently unaffected by a magnetic field. Careful measurements show that of the 62 lines of complex Zeeman effect which satisfy proposition 1, 70 per cent can be reduced to symmetrical triplets and thus satisfy proposition 2. Similar results are obtained for the iron spectrum.

APPARATUS

To carry out the experimental work a large concave grating of 21-foot focus, 20,000 lines per inch, and a 6-inch ruled surface was used. The form of mounting is that of Paschen at Tübingen, the grating and slit being fixed. The spectrum can be examined from λ 2000 in the first order to λ 7000 in the second; thus photographs may be taken simultaneously in both orders. The grating remains

[1] *Loc. cit.*

in perfect adjustment for long periods of time, it now having been used for more than a year without readjustment. The temperature of the grating-room remains so constant that exposures of several hours are successfully made.

The source of light was an electric spark. A condenser of variable capacity from 0.01 to 0.06 m.f. was used in the oscillatory circuit, the spark length being about half a centimeter. The use of self-induction was avoided.

The earlier experiments were made with an electromagnet giving a field of 14,000 gausses with a gap of 8 mm. A Weiss electromagnet is now in use which gives a field of 23,800 gausses for a gap of 12 mm. This is especially convenient, since it can be rotated so that the spectrum may be examined along the lines of force or at right angles to them.

At right angles to the field a nicol prism is used to separate the *p*- and *s*-components. Along the field a Fresnel rhomb and a nicol prism are used to separate the two kinds of circularly polarized components. All of these components, as well as comparison spectra, may be obtained on the same negative by means of a slit diaphragm placed on a separate holder immediately in front of the plate-holder.

SUMMARY

Some work on series spectra, which the writer hopes to publish soon, led him to suspect that certain spectral lines are in reality close doublets, and that therefore they may be expected to show a complex Zeeman effect consisting of two *p*- and four *s*-components. This is indeed found to be the case. Conversely, the writer conceived the idea, independently of other investigators, that complex Zeeman effects occur because spectral lines which give rise to more than three Zeeman components are themselves complex in structure, and that the number of components in such a parent line is exactly equal to the number of *p*-components which it shows when placed in a magnetic field.

Investigation has shown that, in the titanium spectrum between λ 5050 and λ 3230, 62 out of 67 lines of complex Zeeman effect have twice as many *s*- as *p*-components; and that at least 70 per

cent of these may be reduced to symmetrical Zeeman triplets, one for each p-component. In each complex type the reduced triplets are all of the same interval, which varies, however, from type to type in the same manner as for ordinary Zeeman triplets. Further study led to the prediction that in certain cases there are present one or more s-components which should appear unpolarized when viewed parallel to the field. This has been verified experimentally.

Several important questions still remain to be solved. One is the reason for the unsymmetrical distribution of the intensities which frequently occurs in the components of the reduced triplets. Another is that of the separation of the p-components when the magnetic field is applied. A third is that of the abnormal Zeeman intervals. But the same fundamental cause which gives rise to ordinary abnormal Zeeman triplets is probably responsible for the abnormal intervals of the Zeeman triplets to which the complex types may be reduced.

In conclusion, the writer wishes to recognize the valuable help of Professor Theodore Lyman, of Harvard University, under whose supervision this work has been done. His many suggestions, particularly as to the best method of presenting the subject, have been invaluable.

JEFFERSON PHYSICAL LABORATORY
CAMBRIDGE MASS.
October 1916

THE NATURE OF THE CONSTANT-ERROR TERM *K*

SECOND PAPER

By C. D. PERRINE

In my first paper on this subject[1] the question was raised whether the term *K*, found in the solution for solar motion from radial velocities, was in fact constant. Some later evidence makes it desirable to consider this matter again. Unfortunately this is a question which cannot be answered directly by obtaining positive evidence as to whether there is or is not a pressure-effect in the atmospheres of any of the stars of a nature to cause displacements of the spectral lines, but must be attacked more or less indirectly, at least for the present. The solar motion, general systematic motion of the stars, and constant error are so linked together that it appears impossible completely to separate any one. It may be possible later to obtain a consistent apex for the solar motion from proper motions. The velocity of the system, which is really the important factor, would still be in doubt, however.

It may be pointed out in further explanation that in the solutions of the radial velocities for apex and velocity of solar motion there is usually an excess which cannot be represented by the rigid spherical relations existing between the coefficients of the three unknowns in the equations of condition. From the nature of the observed velocities a perfect solution is scarcely to be expected. In the spectral classes B, K, and M these excesses are large and have the same algebraic sign. The introduction into the equations of a constant term for each star or similar one independent of the spherical relations referred to will absorb the excess found. The evidence for such a constant-error term appears to rest wholly upon the excesses mentioned above in solutions for solar motion.

In my first paper the results were derived by freeing the stars from the solar motion on the basis of common apex and velocity, in that case approximately those derived from the same system

[1] *Astrophysical Journal*, 43, 286, 1916.

of stars. While this is probably justifiable in fact as evidence, in this particular instance it does not seem as logical or strong as to make separate solutions in each spectral class for apex and velocity of solar motion and K term simultaneously. The desirability of doing this was accentuated by the finding of differences in the position of the apex in the different spectral classes. Such results are, then, the best representation which can be obtained from the data, and the values of the constant-error term are directly comparable with values derived in a similar way from all of the stars.

The finding of systematic differences in the proper motion of the stars of Class B in the northern and southern sky, as well as of apparent peculiarities in the radial velocities in these two regions, led to a more careful investigation of these regions, in connection with the question of the constant error.

In a paper[1] giving the results of separate determinations from northern and southern stars and according to proper motions, the values of K were uniformly small when obtained from these two regions separately. This again is not entirely independent evidence. If, however, it can be shown that certain groups of stars or those in certain regions have produced the excesses giving rise to the K term, in general solutions from all parts of the sky, such evidence is independent and of great weight, as such values have resulted in exactly the same way as the original (large) values of K.

It was found that there were large excesses of positive velocities and of stars in the southern sky in the classes B, K, and M (those yielding considerable values for K), whereas there was practical equality in the classes A, F, and G.

Further investigation by correcting the different spectral classes by *regions* for a common solar motion of 19.5 km toward $\alpha = 270°$, $\delta = +30°$, showed that there are large systematic velocities in regions of considerable size, that the same general preferences of these excesses are found in all of the spectral classes for the northern sky and generally for the southern sky also in the region from right ascension 18^h to 4^h, but that in the southern sky from approximately 4^h to 18^h there is a radical difference. In this region large negative velocities are all but universal in the types A, F, and G,

[1] *Astrophysical Journal*, 44, 103, 1916.

whereas in the types B, K, and M, negative velocities are either almost entirely wanting or they are restricted to smaller regions. The residual velocities of these type areas are of considerable size and uniformity. The positive residuals appear to increase from the B stars to the K and M stars. They appear to lie in general on great circles, but not parallel to the Milky Way. These peculiarities will be dealt with in a separate paper. For our present purpose it seems only necessary to say that their size and the general correctness of the assumption as to elimination of solar motion make it seem unlikely that they are fictitious. This conclusion is further strengthened by the fact that the very high velocities of all spectral classes (which were not used in these solutions) show somewhat similar distributions. It was found that in these regions the large positive velocities usually fell above a fairly well-defined limit and could be approximately isolated by that means. The criterion was adopted, therefore, with the lower included limits as follows:

B ....+ 6 km
K ....+ 11
M....+ 15

Only two or three exceptions to these limits occurred, and these were either accidental or, in one case, due to the fact that a region was somewhat isolated. The regions rejected are given in Table I. v is the residual velocity after correction for the solar motion toward $\alpha = 270°$, $\delta = +30°$. The B stars were corrected for a solar velocity of 20 km, and the K and M stars for 19.5 km. These regions (averaging about a third of the total) were then rejected and solutions made from the remaining ones for the apex and velocity of solar motion and constant error. The results are given in Table II.

The effect of omitting these large positive regions in these three classes is very marked. The constant-error term has practically disappeared and the values of the solar motion have been brought into much better agreement with the values from Classes A, F, and G. There has been an improvement also in the agreement of the declinations of the apex.

It may be argued that the use of a given set of elements for the solar motion and the rejection of those most divergent will of necessity tend to a closer agreement with the adopted elements. While

TABLE I

3.^Mo AND FAINTER

Spectral Class	α	δ	No. of Stars	Obs. *V.*	*v*
				km	km
	4^h16^m	+64°	2	+ 6	+ 8
	16 47	+43	4	−12	+ 7
	18 12	+23	4	−10	+10
	22 31	+14	3	+ 6	+14
	5 54	− 9	14	+25	+ 7
B	6 0	−68	2	+22	+ 6
	7 3	−25	4	+33	+14
	9 11	−63	4	+23	+ 9
	10 0	− 7	2	+19	+ 9
	13 6	−47	15	+12	+ 9
	14 46	−45	9	+ 6	+ 6
	1 15	+15	6	+20	+17
	4 30	+16	6	+23	+11
	8 58	+16	7	+23	+14
	8 58	+74	2	+ 4	+10
	9 0	+40	3	+17	+14
	10 43	+ 6	2	+16	+12
	19 9	+68	3	+21	+36
K	20 57	+26	4	0	+15
	3 17	−70	4	+27	+14
	5 9	−69	5	+29	+14
	6 50	−69	5	+31	+16
	7 4	−44	8	+34	+16
	8 36	−68	4	+26	+12
	9 3	−18	7	+30	+16
	4 52	+40	3	+23	+17
	10 32	+16	2	+20	+17
	22 26	+53	2	+ 9	+21
	3 28	−17	2	+45	+30
M	7 2	−46	2	+38	+20
	8 15	−27	2	+32	+15
	11 28	−31	1	+22	+15
	13 8	− 8	2	+18	+22
	19 56	−28	1	+14	+22
	20 50	−19	3	+ 8	+16

this is undoubtedly true, a more important consideration would seem to be the proportion of such rejections and the nature of the observations omitted. In the present case, the regions (and individual stars) rejected constitute a third or less of the total in

each of the three classes. The residuals in these rejected regions after correction for solar motion, in each class, show considerable uniformity in size, an increase from class to class, and, perhaps most important of all, average from two to four times the size of the constant error found from the solutions. A conclusion which rests upon only one-third of the data can scarcely be considered as established. No one would think, for example, of proposing as characteristics an increase of velocity with increasing age, or a magnitude-velocity equation, or almost any other, if it were

TABLE II*

	A	D	$V_\odot$	K	No. of Stars	No. of Equations
			km	km		
B {3.0 and fainter—all..	276.0	+29.6	−20.3	+4.0	193	31
B {Rejecting 11 regions.	274.1	+28.5	−19.4	+1.2	130	20
K {3.0 and fainter—all..	274.2	+25.6	−20.7	+3.6	391	63
K {Rejecting 14 regions.	268.1	+30.5	−17.7	+0.1	325	49
M {3.0 and fainter—all..	269.7	+31.7	−22.9	+4.0	65	31
M {Rejecting 10 regions.	265.4	+31.9	−18.8	−3.2	45	21
2.9 and brighter— All spectral classes ...	258.0	+41.5	−18.9	+1.0	110	33

*The results in this table depend upon Campbell's catalogues of radial velocities.
Velocities of 50 km and over in class K, and 40 km and over in class M, were omitted.
Each equation of condition is the mean of all the stars in a region 2^h in right ascension by 30° in declination.

known that two-thirds of the data failed to show it. The constant error in question seems to be no exception. I see no reason why this conclusion as to the sufficiency of evidence should not apply to the question of a constant error, whether there are any peculiarities of distribution or not—that even without any other evidence we should be fully justified in concluding that the excesses observed cannot be ascribed to a constant error. In any case the burden of proof is certainly upon the constant-error explanation. But there are other points bearing upon its reality as a constant error:

1. There is undoubted system in the distribution of these large residual velocities.

2. The assumption of a constant error does not reconcile the values of the solar velocity derived from the different spectral classes.

3. The brighter stars by themselves show only a very small excess although more than half belong to the spectral classes B, K, and M.

4. There is strong evidence that the general motions of the stars of different spectral and other classifications are different. It therefore seems not improbable that the observed residuals are nothing more than motion and may be related to these differences of general motion.

5. It seems more natural and reasonable to ascribe the residuals observed to motion, which we know exists, than to a physical cause of which there is no direct evidence whatever, even if there were considerable uniformity in those excesses.

The system of 1300 stars upon which these investigations are based yields an apex differing 5° from that used in the elimination of the solar motion for the purpose of isolating the peculiar regions. The application of the correction for this difference of apex and the constant error of +1.9 km from all of these stars would not obliterate the large values which have been rejected—those peculiarities would stand out almost as strikingly as they do without such corrections. The very fact that the values of the solar velocity and of the *K* term from all of the stars are a compromise of the values found from the spectral classes separately is further evidence that constancy is not necessarily implied but must be proven quite independently of the solutions.

The foregoing results appear to be sufficient, without discussing other evidence, to justify the conclusion that the observed residuals in the solutions for solar motion from radial velocities are not sufficiently uniform to be treated as a constant error, but are much better explained by motion.

OBSERVATORIO NACIONAL ARGENTINO
CÓRDOBA
August 2, 1916

ON THE ORBITS OF THE SPECTROSCOPIC BINARIES α ORIONIS AND α SCORPII

By JOSEPH LUNT

These two stars α Orionis and α Scorpii have the peculiarity of showing the smallest range of variation of radial velocity of any of the spectroscopic binaries for which orbits have been derived, the semi-amplitude of the velocity-curve being only 2.45 km per sec. in the case of α Orionis according to Bottlinger's elements,[1] and 2.12 km according to Halm's elements[2] in the case of α Scorpii. As these quantities are comparable with the accidental errors of the observations, it is of interest and importance to examine the evidence on which the orbits are based and to supplement it by measures of a large number of plates taken at the Cape, of which duplicate measures have, in most cases, been made by Halm and myself. It is desirable to call attention to the fact that in spite of the large number of determinations of velocity of these two stars which have been made, it is still necessary to continue the accumulation of material, as the orbits—particularly in the case of α Scorpii—are obviously in need of revision.

Ludendorff called Bottlinger's attention to the similarity of the orbits derived for these two stars, and the latter states that they are the only spectroscopic binaries of Class III*a* of which the elements are known.

The case of α Orionis is of further interest from the fact that Bottlinger has attempted to connect the variations of radial velocity with the variations of brightness of the star and suggests a relationship similar to that found to exist in the case of stars of the δ Cephei type, noting that the period and type of spectrum are very different.[3] Charles P. Olivier,[4] from a discussion of 293 observations of brightness extending over a period of nearly eleven

[1] *Astronomische Nachrichten*, 187, 33, 1911.

[2] *Annals of the Cape Observatory*, 10, Part III, 56*c*.

[3] *Astronomische Nachrichten*, 187, 33, 1911.

[4] *Ibid.*, 194, 81, 1913.

years, from 1901 to 1912, concluded that "it was quite impossible to find any regularities in the light-changes." On plotting his observations together with the velocity-determinations and theoretical velocity-curve, no connection appears to exist, and the observations negative the idea of a relationship similar to that found in the case of δ Cephei and other analogous stars.

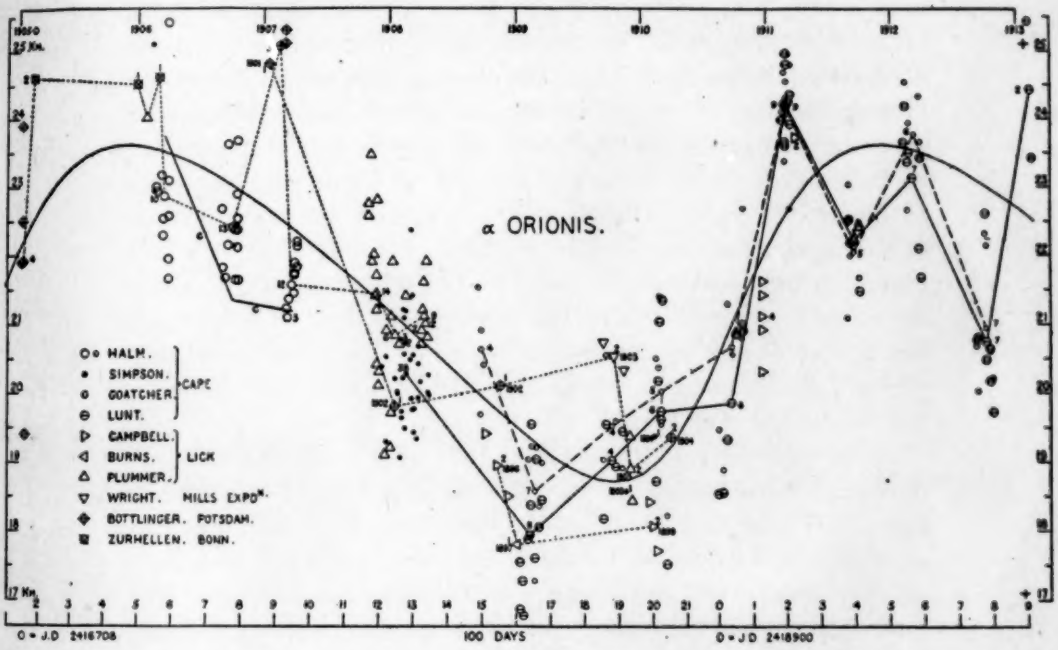


FIG. 1

In order to exhibit the facts as regards the determination of the orbits in a graphic manner, two charts have been prepared which clearly show the present position of the problem and indicate what further work is necessary in order to arrive at a satisfactory solution. The original charts were made on millimeter-squared paper on a large scale, viz., 1 cm=0.2 km, 1 cm=40 days, and were reduced photographically to a convenient size for the pages of the *Journal*.

α ORIONIS

In Fig. 1 the sinuous line represents the theoretical velocity-curve derived from the elements by Bottlinger. The individual

observations on which the orbit is based are shown, and the means of each group are connected by the fine broken line, the number of plates in each group being denoted by numerals at each of the connected points. The observations of different observers are differentiated in the diagram and they are divided into three periods, viz., 1896–1898, 1901–1904, and 1905–1913. The observations in the first two periods are denoted by dates at the mean points in the diagram. To the rest of the observations the dates at the head of the diagram and the 100-day scale at the foot of the diagram apply. Included in the additional observations here given are 38 measures by Simpson[1]—published after the appearance of Bottlinger's paper—from plates taken at about the same time as Plummer's series. Halm's measures, mainly duplicating those of the same plates measured by me but using a different standard plate, are indicated by the coarse broken line from 1908 to 1912, my measures being indicated by a continuous line. The continuous line 1906–1907 represents the means of Goatcher's measures,[2] on the Zeiss comparator, of the same plates measured by Halm on the Hartmann comparator.

It is apparent from the diagram that no point on the velocity-curve can be satisfactorily fixed by the measures of only one or two plates, and that Bottlinger's determinations of the important maximum and minimum points of the curve are weak, inasmuch as they depend on scanty and non-homogeneous material. The later observations, depending on a larger number of plates, indicate that the minimum is earlier and lower than Bottlinger's orbit demands and that the maximum is earlier and higher. Whether the pronounced departures from simple orbital motion which are so strikingly shown in the duplicate measures of the same plates by Halm and myself are real, and analogous to those regarded as established by Campbell in the case of ζ Geminorum, or whether they are instrumental in origin, is a point which can be settled only by further observations, preferably at different observatories with different instruments.

It is evident from Bottlinger's measures of the Potsdam plates, from Plummer's measures of the Lick plates, and from measures of

[1] *Annals of the Cape Observatory*, 10, Part III, 91*c*. [2] *Ibid.*, 10, Part I, 54.

the Cape plates by various observers that isolated observations are of little value in determining orbits of binaries showing such small amplitude, but groups of ten or twelve plates a few months apart, particularly at maximum and minimum, are much needed. Plummer's series of measures of Lick plates and Simpson's of Cape plates at about the same epoch afford a good opportunity of comparing results and indicate a difference Lick−Cape of 0.64 km.

In the period from December 17, 1907, to April 23, 1908:

Plummer 17 plates	= 20.88 km	Mean phase 1291^d
Simpson 36 "	= 20.24 "	" " 1286^d

In determining orbits of stars such as these, with such a small variation in velocity, there is some danger in combining heterogeneous material from different observatories when corrections have to be applied to bring them into agreement.

Campbell records that fifteen observations by Belopolsky at Pulkowo in 1898 and 1899 on the average are 6 km above his values for ζ Geminorum. Bottlinger applied a correction of 2 km to the Potsdam and Bonn plates, but Küstner and Zurhellen, from observations of the starlike mountain peaks near the lunar terminator, arrive at a correction of only 1 km. Zurhellen's values would therefore be 1 km higher than is shown in the diagram. It is not considered advisable to attempt to revise Bottlinger's elements until further observations are secured.

The observations are given in Table I.

The measures of forty-four plates by Simpson are published in the *Annals of the Cape Observatory*, **10**, Part I, 54. The six measures of three Lick plates, 1910–1911, are published in *Lick Observatory Bulletin*, **6**, 144. One plate taken on December 21, 1910, was measured independently by Miss Hobe and Messrs. Burns, Wilson, and Young, and two plates taken on March 27 and 28, 1911, were measured by Miss Hobe. In the diagram these are marked as Campbell plates.

The Cape measures were made on a Hartmann spectrocomparator. Halm used plate 3453 of α Orionis as standard plate, the shift of which has been taken provisionally as +45.81 km, the mean of three comparisons with the plate 2145 of α Tauri used as

TABLE I

α Orionis

Plate No.	Date	Phase*	Radial Velocity		L.—H.
			Lunt	Halm	
			km	km	km
1901	1908 Sept. 14	1491d	.....	+21.52	
1909	21	98	.....	19.68	
1913	22	99	.....	20.90	
1922	30	1507	.....	20.40	
	Mean, 4 plates....	1499	.....	20.62	
2117	1909 Jan. 13	1612	+17.55	.....	
2123	14	13	16.87	.....	
2146	21	20	17.28	.....	
2159	23	22	16.77	.....	
2207	Feb. 6	36	17.88	19.03	
2226	15	45	18.37	17.92	
2229	16	46	19.53	19.22	
2244	25	55	17.60	17.28	
2258	Mar. 2	60	19.03	19.20	
2271	10	68	18.05	18.35	
2282	19	77	18.44	18.98	
	Mean, 11 plates....	1641	17.94	.....	
	Mean, 7 plates....	1655	.....	18.57	−0.63
2501	1909 Sept. 15	1857	18.17	19.02	
2515	22	64	19.53	20.54	
2542	Oct. 20	92	18.92	19.58	
2566	Nov. 9	1912	19.43	18.90	
	Mean, 4 plates....	1881	19.01	19.51	−0.50
2675	1910 Feb. 15	2010	18.71	20.49	
2683	22	17	20.16	20.36	
2689	25	20	21.02	21.39	
2694	Mar. 1	24	19.60	19.53	
2705	5	28	21.33	.....	
2707	19	42	17.51	18.21	
	Mean, 5 plates.....	2023	.....	20.00	
	Mean, 6 plates.....	2024	19.72	.....	−0.28
2796	1910 Aug. 19	3	18.52	19.45	
2806	Sept. 2	17	18.54	18.87	
2818	13	28	19.31	21.28	
2840	Oct. 6	51	20.84	20.89	
2852	25	70	20.98	20.79	
2855	28	73	20.89	22.67	
	Mean, 6 plates.....	40d	+19.85	+20.66	−0.81

*Zero phase is taken as J. D. 2416708 = 1904 August 15, and the period as 2192 days.

TABLE I—*Continued*

Plate No.	Date	Phase	Radial Velocity		L.—H.
			Lunt	Halm	
			km	km	km
3008	1911 Feb. 16	184^d	+23.97	+24.09	
3021	22	190	24.20	24.67	
3024	24	192	23.64	24.03	
3027	25	193	23.59	23.36	
3029	27	195	24.95	24.54	
3032	Mar. 1	197	24.79	24.29	
3042	8	204	24.24	24.94	
3048	11	207	23.90	22.67	
3049	13	209	24.34	24.78	
	Mean, 9 plates	197	24.18	24.15	+0.03
3260	1911 Aug. 28	377	22.49	21.08	
3265	31	380	22.51	23.02	
3268	Sept. 5	385	22.17	21.98	
3283	29	409	22.41	22.47	
3290	Oct. 2	412	21.47	21.66	
	Mean, 5 plates	393	22.21	22.04	+0.17
3453	1912 Feb. 6	539	23.63	Standard	
3470	12	545	24.17	24.53	
3480	17	550	23.35	22.65	
3489	21	554	23.49	23.92	
3499	23	556†	26.94†	27.36†	
3547	Mar. 23	585	22.09	24.32	
3553	25	587	23.41	23.65	
3567	29	591	21.67	23.45	
	Mean, 7 plates	564	23.12	.....	
	Mean, 6 plates	569	.....	23.75	−0.63
3796	1912 Sept. 6	752	20.77	20.67	
3803	10	756	.....	20.02	
3825	29	775	20.80	22.31	
3830	30	776	22.60	.....	
3835	Oct. 4	780	20.48	22.14	
3847	14	790	20.64	20.60	
3849	15	791	20.17	20.66	
3864	23	799	19.72	20.21	
	Mean, 7 plates	780	20.74	.....	
	Mean, 7 plates	778	.....	+20.94	−0.20
3969	1913 Feb. 3	902	25.43	.....	
3973	14	913	23.42	.....	
	Mean, 2 plates	907^d	+24.43	.....	

† Plate rejected.

standard plate by Lunt. The plate of α Tauri has a shift of +77.38 km, as determined by comparison with twenty solar plates. Halm's measures may require a further correction when the shift of his standard plate is better determined.

α SCORPII

The diagram of velocities, Fig. 2, is self-explanatory and the same general remarks apply as in the case of α Orionis. Halm's

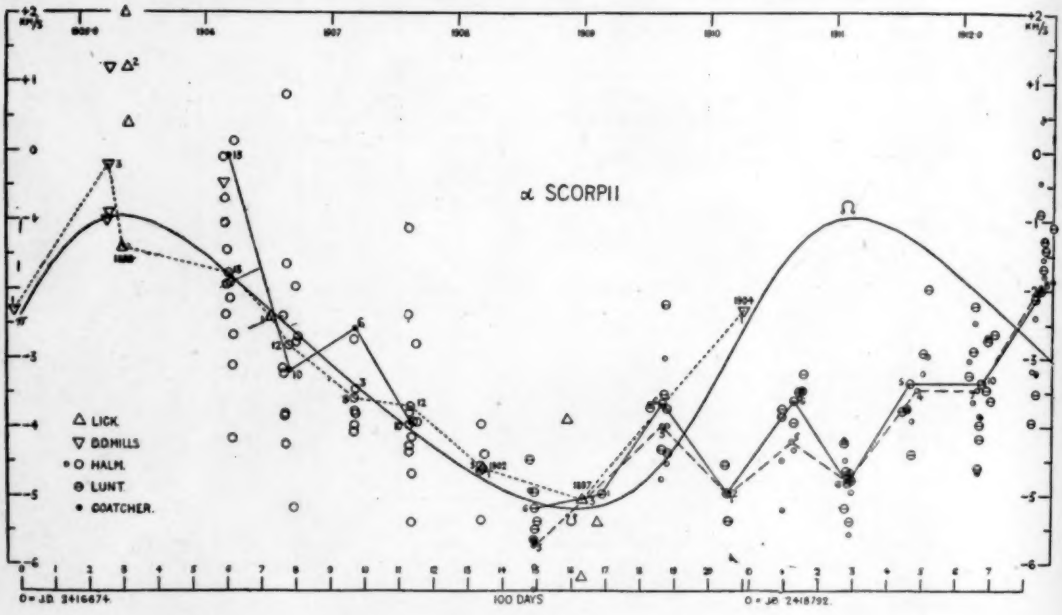


FIG. 2

published measures are confined to a part of the downward branch of the velocity-curve covering a period of a little over two years and comprising fifty plates. The individual measures of groups of twelve or thirteen plates show fluctuations which in range exceed the total variation of velocity required by the orbit derived, and show that measures of isolated plates are of little or no value in fixing a point on the velocity-curve. The remaining and major part of the velocity-curve depends on only nine plates, five secured at Lick and four by the Southern Mills Expedition at Santiago. The apex of the curve depends on approximate measures of three

TABLE II

α SCORPII

PLATE NO.	DATE	PHASE*	RADIAL VELOCITY		L.–H.
			Lunt	Halm	
			km	km	
1786	1908 July 30	1479d	−4.48	−4.93	
1807	Aug. 10	1490	5.65	6.38	
1813	11	91	5.18	5.70	
1816	12	92	4.95	5.65	
1822	13	93	5.50	6.05	
1846	21	1501	5.38	Standard	
	Mean, 5 plates	1489	.....	5.74	
	Mean, 6 plates	1491	5.19	.....	+0.55
2247	1909 Feb. 25	1689	5.38†	.....	
2414	July 15	1829	3.73	3.67	
2456	Aug. 17	62	4.33	4.76	
2464	26	71	3.53	.....	
2468	27	72	2.22	3.00	
2475	Sept. 2	78	3.74	4.53	
2481	3	79	4.38	3.99	
	Mean, 5 plates	1864	.....	3.99	
	Mean, 6 plates	1865	3.66	.....	+0.33
2682	1910 Feb. 20	2049	4.56	.....	
2698	Mar. 1	2058	5.38	4.97	·0.00
	Mean, 2 plates	2054	4.97	.....	
2774	1910 Aug. 2	94	3.85	4.49	
2777	3	95	3.74	5.22	
2809	Sept. 6	129	3.94	4.33	
2825	21	144	3.49	3.56	
2831	29	152	3.48	3.47	
2835	Oct. 4	157	3.23	.....	
	Mean, 5 plates	123	.....	4.21	
	Mean, 6 plates	129	3.62	.....	+0.59
2969	1911 Jan. 31	276	4.24	4.19	
2975	Feb. 2	278	5.19	4.49	
2990	6	282	4.65	4.72	
3003	15	291	5.39	5.57	
3010	16	292	4.68	4.74	
3014	20	296	4.78	4.95	
	Mean, 6 plates	286d	−4.82	−4.78	−0.04

* Zero phase taken as J.D. 2416674 = 1904 July 12, and the period as 2118 days.
† Radial velocity 4.98 in diagram in error.

TABLE II—*Continued*

Plate No.	Date	Phase	Radial Velocity		L.–H.
			Lunt	Halm	
			km	km	km
3231	1911 July 18	444[d]	−3.77	.....	
3240	31	457	3.73	−3.74	
3245	Aug. 14	471	4.40	3.91	
3278	Sept. 15	503	2.92	3.22	
3280	29	517	1.99	2.97	
	Mean, 5 plates	478‡	3.36	.....	
	Mean, 4 plates	487	.....	3.46	+0.10
3443	1912 Jan. 29	639	3.25	3.04	
3462	Feb. 8	649	2.89	3.64	
3473	12	653	2.23	2.48	
3495	22	663	4.60	4.67	
3505	27	668	4.17	3.84	
3508	28	669	3.97	3.85	
3542	Mar. 17	687	3.46	.....	
3545	21	691	2.74	2.68	
3568	31	701	3.62	.....	
3570	April 8	709	2.64	.....	
	Mean, 7 plates	662	.....	3.46	
	Mean, 10 plates	673	3.36	.....	+0.10
3739	1912 July 29	821	3.94	3.18	
3756	Aug. 7	830	2.12	2.40	
3763	10	833	3.51	3.21	
3768	19	842	0.90	0.45	
3782	30	853	1.69	1.55	
3789	Sept. 2	856	1.29	1.80	
3792	6	860	1.42	1.25	
3827	30	884	1.09	1.87	
	Mean, 8 plates	847[d]	−2.00	−1.96	−0.04

‡ Phase shown as 465 days in error.

Santiago plates—mean, −0.2 km per sec.—and one Lick plate belonging to a previous period. Two other Lick plates which place the observed maximum at +1.2 km have been left out of account, but if these are included and the revised[1] values of the three Santiago plates are used, the mean of the five 1905 plates is 1.09 km above the theoretical maximum of the curve, and the two Lick plates place it 2.16 km above the curve, leaving the maximum very uncertain. The minimum of the curve depends on three Lick plates taken in 1897, and the period depends mainly on the

[1] *Publications of the Lick Observatory*, **9**, 257.

coincidence in velocity between one Lick plate at 1902.4 and five Cape plates at 1908.2, giving a period of 5.8 years. On the ascending branch of the curve there is only a single Lick plate.

The velocity-curve derived from measures of fifty plates from July, 1908 to October, 1912—subsequent to the first series measured by Halm—does not follow the course prescribed for it by the orbit, but indicates that the maximum of the curve had not then been reached. It indicates that the period must be lengthened very considerably and that a satisfactory orbit cannot be derived until the observations are extended to give a determination of the apex of the curve. In view of the small range of variability it is desirable to secure observations in groups of plates, say ten to twelve, at intervals of four to six months, and that the observations should be duplicated at different observatories.

The observations are given in Table II.

Halm used Plate 1846 of α Scorpii as standard plate.

Lunt used Plate 2145 of α Tauri as standard plate.

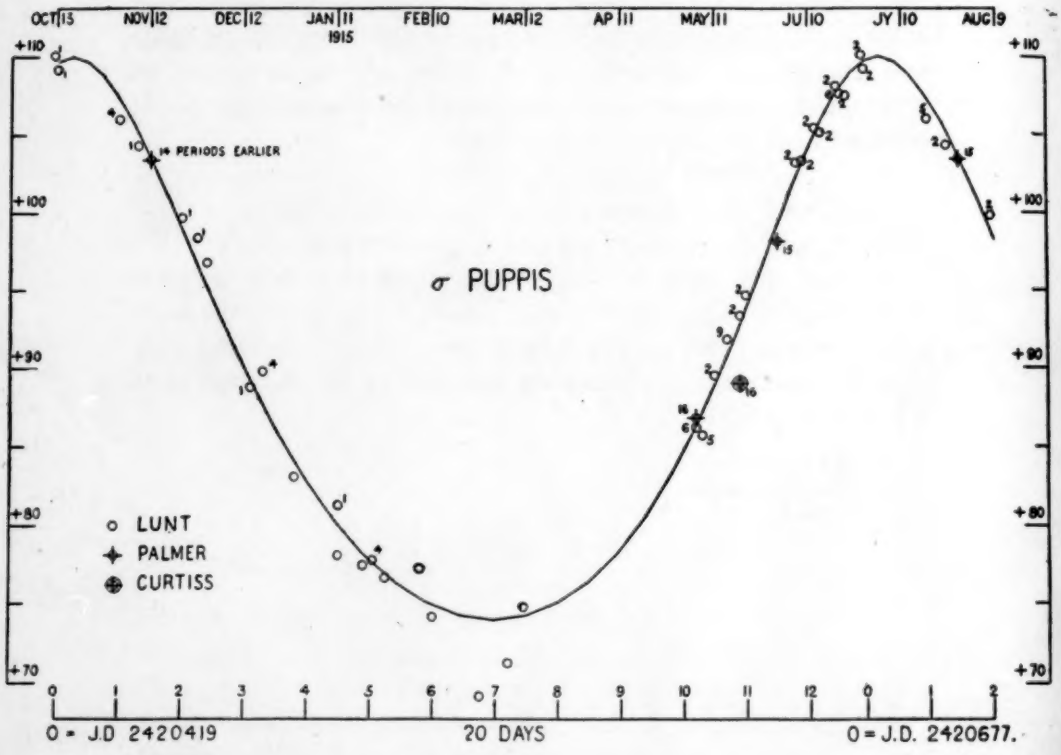
Shift of 1846 taken as +23.43 km, the mean of four comparisons with plate 2145. Shift of 2145 taken as +77.38 km, the mean of comparisons with twenty solar plates. Halm's measures may require a further correction when the shift of his standard plate is better determined.

ROYAL OBSERVATORY
CAPE OF GOOD HOPE
April 1916

ON THE ORBIT OF THE SPECTROSCOPIC BINARY σ PUPPIS

By JOSEPH LUNT

The variable radial velocity of this star (for which $\alpha = 7^h 26^m.1$, $\delta = -43°6'$ [1900]; magnitude, 2.99; type, K5M) was announced in *Lick Observatory Bulletin*, No. 75 (3, 111) as detected by Dr. Palmer



in measures of 4 plates taken by the D. O. Mills Expedition, Santiago, in 1904.

Thirty-three plates of the spectrum of this star were obtained at the Cape between January 4, 1909, and March 11, 1915. The measures of these confirm the variability, and suffice for a prelim-

inary determination of the orbit, which appears not to have been published previously.

Table I gives the observations, in which the four Santiago plates[1] are included. The diagram shows the observations compared with the theoretical velocity-curve computed from the following elements derived by graphical methods:

TABLE I

Plate No.	Date	Radial Velocity	Plate No.	Date	Radial Velocity
		km			km
	1904 Jan. 15*	+ 86.8	4207.....	1914 Jan. 22	+107.5
	29*	89.0	4215.....	27	110.1
	Oct. 25*	98.1†	4216.....	28	109.2
	Dec. 22*	103.4	4236.....	Feb. 23	104.3
2101.....	1909 Jan. 4	91.8	4250.....	Mar. 9	99.7
2991.....	1911 Feb. 7	86.2	4257.....	14	98.4
3082.....	Mar. 25	107.6	4273.....	31	88.8
3323.....	Oct. 25	85.7	4296.....	Apr. 28	78.1
3406.....	1912 Jan. 5	106.0	4455.....	Nov. 30	96.5
3485.....	Feb. 20	89.8	4468.....	Dec. 28	83.1
3556.....	Mar. 26	77.8	4471.....	1915 Jan. 11	81.3
4172.....	1913 Dec. 11	89.5	4476.....	19	77.6
4180.....	19	93.3	4482.....	26	76.7
4182.....	21	94.6	4489.....	Feb. 4	77.0
4190.....	1914 Jan. 6	103.2	4494.....	10	74.2
4193.....	8	103.3	4506.....	25	69.2
4199.....	12	105.4	4513.....	Mar. 6	71.3
4200.....	14	105.1	4519.....	11	74.8
4206.....	19	108.1			

* Santiago plates.

† Correction +1.1 km applied.

$$P = 258 \text{ days}$$
$$T = \text{J.D. } 2420419$$
$$\omega = 350^\circ$$
$$e = 0.2$$
$$K = 18.0 \text{ km}$$
$$a \sin i = 62{,}570{,}000 \text{ km}$$
$$V_o = +88.5 \text{ km}$$

[1] In *Publications of the Lick Observatory*, 9, 148, the following revised values of these velocities are given:

+85.37 +89.00 +96.95 wt ½ +102.79 km

The figures placed beside the observations represent the number of periods which must be added to the date of observation to bring it up to the date given at the head of the diagram.

The determination of the minimum of the velocity-curve is weak and includes two doubtful observations, but as no opportunity occurs to revise it until March-May 1917, it seems desirable to publish the data already secured.

In Table II the observed velocities, with the values derived from the theoretical curve, and their differences, are given in order of phase.

TABLE II

OBSERVATIONS OF σ PUPPIS ARRANGED IN ORDER OF PHASE

Phase	O.	C.	O.–C.	Phase	O.	C.	O.–C.
	km	km	km		km	km	km
0^d....	+109.2	+109.7	−0.5	149^d....	+ 74.8	+ 74.2	+0.6
20.....	106.0	107.3	−1.3	204.....	86.8*	86.2	+0.6
26.....	104.3	105.3	−1.0	204.....	86.2	86.2	0.0
30.....	103.4*	103.5	−0.1	206.....	85.7	87.0	−1.3
40.....	99.7	99.2	+0.5	210.....	89.5	88.9	+0.6
45.....	98.4	96.8	+1.6	214.....	91.8	90.6	+1.2
48.....	96.5	95.4	+1.1	218.....	89.0*	92.6	−3.6
62.....	88.8	89.2	−0.4	218.....	93.3	92.6	+0.7
66.....	89.8	87.7	+2.1	220.....	94.6	93.6	+1.0
76.....	83.1	84.1	−1.0	230.....	98.1*	99.3	−1.2†
90.....	78.1	80.0	−1.9	236.....	103.2	102.2	+1.0
90.....	81.3	80.0	+1.3	238.....	103.3	103.2	+0.1
98.....	77.6	78.2	−0.6	242.....	105.4	105.0	+0.4
101.....	77.8	77.6	+0.2	244.....	105.1	105.9	−0.8
105.....	76.7	76.9	−0.2	249.....	108.1	107.7	+0.4
114.....	77.0	75.6	+1.4	250.....	107.6	108.0	−0.4
120.....	74.2	75.0	−0.8	252.....	107.5	108.5	−1.0
135.....	69.2	74.0	−4.8	257.....	110.1	109.5	+0.6
144.....	71.3	74.0	−2.7	258 or 0^d	109.2	109.7	−0.5

* Santiago observations.

† Correction +1.1 km applied.

The measures were made on the Hartmann spectrocomparator, using Plate 2145, of α Tauri, as standard.

ROYAL OBSERVATORY
CAPE OF GOOD HOPE
April 1916

ERRATA

Vol. 42, December 1915, "The Elements of the Eclipsing Systems TV, TW, TX Cassiopeiae and T Leonis Minoris," by R. J. McDiarmid:

Page 422, last line, *for* ak^2 *read* $a_2^2k^2$

" 431, sixth line, *for* c *read* C

" 432, Table VII, the following numerical values should be substituted:

	Uniform	Darkened
a_b	0.285	0.287
b_b	0.271	0.277
a_f	0.301	0.325
b_f	0.285	0.314
ρ_b	0.137	0.127
ρ_f	0.055	0.044

Distance in light-years 1030

This change in dimensions of stars means that the larger star is the fainter in both cases, so that in Fig. 2 the stars are interchanged.

Vol. 43, June 1916, "Researches on Solar Vortices," by Carl Störmer:

Page 348, second equation, *for* ϕ *read* ϕ_0.

" 352, *delete* fourth and fifth lines from bottom.

" 363, *interchange* legends of two parts of Fig. 9.

" 401, second formula, the quantities between the inequality signs are two parts of a single expression. The comma should be replaced by multiplication sign.

Delete last sentence of the second paragraph of Section XIV.

Vol. 44, September 1916, "Some Determinations of the Apex and Velocity of Solar Motion from the Radial Velocities of the Brighter Stars, Including an Apparent Relation to Proper Motion," by C. D. Perrine:

Page 107, Table I, fifth column, $V_\odot$ (North), next to last line, *for* $+2.3$ *read* $+0.7$.

Page 114, Table VII, fifth column, K, next to last line, *for* $+4.3$ *read* $+3.6$.

Vol. 44, September 1916, "The Minimum Radiation Visually Perceptible," by Herbert E. Ives:

Page 127, fifth line, *for* $0.38\times10^{-8}\ \frac{\text{ergs}}{\text{sec. sq. cm.}}$ *read* 0.38×10^{-8} ergs per second.

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